Journal of Industrial Engineering International *Islamic Azad University, South Tehran Branch*
January 2010, Vol. 6, No. 10, 1-15

A model for tracking temperature variation in cold and hot metal working conditions during machining operations

S. A. Oke*

Lecturer, Dep. of Mechanical Engineering, University of Lagos, Nigeria

I. O. Oyedokun

B.Sc., Former Student, Dep. of Mechanical Engineering, University of Lagos, Nigeria

O. S. Momodu

B.Sc., Former Student, Dep. of Mechanical Engineering, University of Lagos, Nigeria

O. G. Akanbi

Lecturer, Dep. of Industrial and Production Engineering, University of Ibadan, Nigeria

F. A. Oyawale

Senior Lecturer, Dep. of Industrial and Production Engineering, University of Ibadan, Nigeria

Abstract

This paper presents a mathematical model that could assist in measuring, monitoring and controlling temperature variation in cold and 'red-hot' metal working conditions of machining. A numerical analysis technique of the temperature distribution, based on the theory of complex applied potential, was carried out using the principles of relationship analysis between the paths of heat supply in Cartesian plane when the heat path supplied to the material is orthogonal. The high level of temperature involved may effectively be predicted if a mathematical relationship that predicts the pattern of temperature distribution in a material is available. A case study example in a machining workshop is given. Simulation experiments are then carried out using Monte Carlo to increase the confidence in decision-making and provide data for significance testing. This was used as an input for testing for significance. Sensitivity analyses were also carried out in order to observe the degree of responsiveness of model parameters to changes in value. In all, five pairs of comparison were carried out among different workpiece materials. There are significant differences between workpiece materials made of steel and copper, copper and zinc, copper and aluminum. However, no significant differences exist in the model behavior of steel and aluminum, steel and zinc. It was observed that parameters are highly sensitive to changes in value. The framework could possibly be applied to milling and surfacing activities in the engineering workshop. This contribution may be helpful to small-scale enterprises that could not afford sophisticated and very expensive facilities.

Keywords: Machining conditions; Temperature distribution; Turning operation; Orthogonal distribution

1. Introduction

Machining, which involves the removal of materials from metal surfaces in order to attain the required shape, texture and size of the component being machined, is one of the important workshop processes that is strongly influenced by temperature conditions of the workpiece. Machining cold and hot workpieces of the same material, at the same cutting speed, and with the same material dimensions may produce different outputs. Thus, tracking temperature distribution at different conditions of coldness and hotness of metal workpieces may provide useful information on the best combination of

* *Corresponding author. E-mail: sa_oke@yahoo.com*

machining conditions necessary to attain the desired properties of components. Although some highly sophisticated techniques and equipment are available in the market for use, yet their prohibitively high cost restrain the small-scale industrial enterprises from their usage. This is particularly the case in developing countries where the gross domestic product (GDP) is low. Despite the inability of small-scale enterprises to afford sophisticated facilities in machine tool production, a large number of these enterprises are still exploring traditional approaches in machining operations. Thus, since the prime objective of SMEs is to optimize profit, there is need for an understanding of the temperature distribution in materials so as to minimize wastes as a result of workpiece breakage during machining. This information could assist management of SMEs in the formulation of policies that would guide choice and usage of materials for component manufacture in the machining workshop. Procedures and systems could also be put in place for the best work practice if the behavior of temperature distribution in materials could easily be predicted.

In machining operations, the removal of metals in the form of chips that involves the reduction of the diameter (thickness) of the metal is referred to as turning operation.

This requires close dimensional accuracy. It is usually performed on machine tools, which include various power-driven machines. These machines operate on either reciprocating or rotatory-type principle: either the tool or the work piece reciprocates or rotates (Oke, *et al.*, 2006).

Turning operation generates a lot of heat on the metal being cut and on the cutting tool because of the relative friction and motion between the cutting tool and the work piece. This invariably makes the chips very hot having high temperature (since temperature is the average measure of heat energy (Varia and Massih, 2002; Kastebo and Cariberg, 2004).

The literature on temperature distribution in metals during machining or other conditions is growing. Fudolig *et al.* (1997) investigated into the numerical analysis of the flow characteristics and temperature distribution in metal beads subjected to transferred-arc plasma impingement.

In another study, Kastebo and Cariberg (2004) investigated into temperature measurements and modeling of heat losses in molten metal distribution systems. Temperature distribution in molten metal flowing in plate-like mould cavity was studied by Matsuda and Ohmi (1981). In addition, finite element and physical simulations of non-steady state metal flow temperature distribution in twin roll strip casting was carried out by Shiomi *et al.* (1995).

In a related study, an investigation was carried out on hydride-induced embrittlement and fracture in metals with emphasis on effect of stress and temperature distribution (Varia and Massih, 2002). In a recent study, numerical analysis of temperature distribution of cold cylindrical metal subject to machining was investigated by Oke *et al.* (2006).

By considering the pool of research carried out on temperature distribution in metals during metal operations, there seems to be a focus on the general determination of temperature distribution (for steel) without guidance on when it could be orthogonal or not. Also, there is limitation to steel as the metal used instead of a general model for most metals. Again, focus has been on only hot conditions, while the temperature considered here could be cold, hot, or "red" hot. In addition, the model predicts if the temperature distribution is orthogonal or not. Till date, no scientific documentation seems to have been made on this topic. Thus, this work closes an important gap in the temperature distribution research. The paper is organized into four sections. The introduction describes the motivation for the study, presents the definition of the problem, the research objective, and the expected contribution of the paper. It also reviews relevant literature on the subject considered. Section two presents the methodology used for investigation in the study. In Section three, a case study is presented in order to increase our understanding and verify the whole model. Hypothetical data is used to illustrate the working of the model from an engineering perspective. Section four presents the conclusion to the study.

2. Methodology

Modelling the pattern of temperature distribution in a material is carried out based on the theory of complex applied potential under a number of assumptions.

These are listed in the relevant subsection under this section. However, this is preceded by the definition of terms used in the current work. Note that the path at which heat is supplied depends on the shape of the workpiece.

2.1 Notations

The notations utilized in the body of this work is varied. However, the list is given as follows:

C_m	Pecific heat capacity of the material M mass,
C = f (x,y,z)	Path of heat supply in cartesian plane,
C = f (r, θ, z)	Path of heat supply in polar plane,
E= h(x,y,z)	Path of temperature distribution,
l	Length of the bar in metres,
R	Initial radius of the bar,
α	Cutting depth,
r	Radius of the cylinder after cut in metres,
$\vec{C}$	Position vector of any point along the path of heat generated,
ρ	Pitch,
θ	Angular displacement,
μ	Coefficient of friction of the material,
T_r	Torque on the material,
W	Weight of the workpiece,
ρ_m	Mass density of the material,
T_1	Temperature after the first turn,
T_2	Temperature after the second turn.

2.2. Assumptions

The following assumptions are made in order to formulate and apply the model.

- The path at which heat is supplied to the tool is assumed to be the path traced by the cutting tool on the workpiece.
- The same measures of heat will be transferred throughout the workpiece because the material is homogenous.
- The path of temperature distribution in the workpiece is orthogonal to the path of heat supplied.

2.3. Mathematical analysis

The starting point in the model analysis is to establish a relationship between h' and f' using the theory complex applied potential (Stroud, 2003). This is presented below in Equation (1):

$$h^1(x,y,z) = \frac{-1}{f^1(x,y,z)} \tag{1}$$

Equation (1) shows the relationship between the path of temperature distribution, $h^1(x,y,z)$, and the path of heat supply in Cartesian plane.

$$E = -\iiint h^1(x,y,z)\,dxdydz \tag{2}$$

This gives the path of temperature distribution in any material when the path of heat supplied to the material is orthogonal. By replacing $h'(x,y,z)$ in Equation (2) by $\frac{-1}{f^1(x,y,z)}$, we obtain:

$$E = -\iiint \frac{dxdydz}{f^i(x,y,z)} \tag{3}$$

Equation (3) gives the relationship between path of temperature distribution in any material when the path of heat supplied is orthogonal to it and the path of heat supplied to the material is in a Cartesian plane.

Figure 1 is a cylindrical shaft with coordinate axes x, y, and z. Now,

$$r = R - \alpha,$$

and

$$\vec{C} = x\hat{i} + y\hat{j} + z\hat{k} \tag{4}$$

where r is the final radius of the cylindrical bar after cutting operation has been performed. $\vec{C}$ gives the position vector of any point along the path of heat generated in the Cartesian plane. Thus,

$$\vec{C} = rCos\,\theta\,\hat{i} + rSin\,\theta\,\hat{j} + z\hat{k} \tag{5}$$

From Equation (5), the position vector of any point along the path of heat generated in the polar plane can be determined. However, let:

$$r = at \tag{6}$$

and

$$z = ct \tag{7}$$

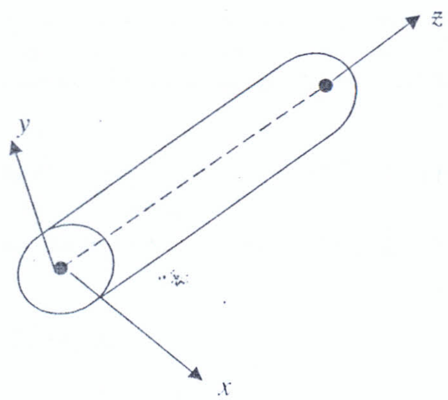

Figure 1. A cylindrical shaft with co-ordinate axes.

These are done to enable us introduce the polar coordinates r and z as functions of t.Therefore, from:

$$C = \sqrt{(r + Cos\,\theta)^2 + (r + Sin\theta)^2 + (z)^2}$$

and substituting for r and z in Equations (6) and (7) respectively, we have:

$$C = \sqrt{(a + Cos\,\theta)^2 + (a + Sin\theta)^2 + (ct)^2} \qquad (8)$$

Expansion of Equation (8) gives:

$$C = [a^2 t^2 (Cos\,\theta^2 + Sin\,\theta^2) + c^2 t^2)]^{0.5} \qquad (9)$$

From the rules of trigonometry, $Cos^2\theta + Sin^2\theta = 1$. By applying this to Equation (8), we note that:

$$C = \sqrt{a^2 t^2 + c^2 t^2} = t\sqrt{a^2 + c^2} \qquad (10)$$

Equation (10) gives the relationship between the path of heat supply and a, c, and t. Then:

$$\frac{dc}{dt} = \sqrt{a^2 + C^2} \qquad (11)$$

which gives the path of heat supply when differentiated with respect to t. Thus, we have

$$[\frac{dE}{dt}] = -\frac{1}{[\frac{dc}{dt}]} \qquad (12)$$

which shows the relationship between the differentials of the path of temperature distribution and the path of heat supply. By substituting Equation (11) into Equation (12), we obtain another expression for the differential of the path of temperature with respect to t as shown in Equation (13), which states that:

$$\frac{dE}{dt} = \frac{-1}{\sqrt{a^2 + c^2}} \qquad (13)$$

By re-arranging Equation (13), we have:

$$dE = \frac{-dt}{\sqrt{a^2 + a^2}} \qquad (14)$$

Integrating both sides:

$$\int dE = -\int \frac{dt}{\sqrt{a^2 + a^2}} = \frac{-t}{\sqrt{a^2 + c^2}} \qquad (15)$$

But $a = \frac{r}{t}$, and $c = \frac{z}{t}$. By substituting for these expressions representing "a" and "c" in Equation (15), we now have:

$$E = \frac{-t}{\sqrt{(\frac{r}{t})^2 + (\frac{z}{t})^2}} = \frac{-t^2}{\sqrt{r^2 + z^2}} \qquad (16)$$

However, $Z = \frac{\rho\theta}{2\pi}$; and $c = \frac{T_r}{\mu W}$. By substituting these variables into Equation (16) we have:

$$E = \frac{-t^2}{\sqrt{[\frac{T_r}{\mu W}]^2 + [\frac{\rho\theta}{2\pi}]^2}} \qquad (17)$$

Knowing that E is the length of curve traced, Equation (17) is a mathematical model that tells us the particular point the temperature generated is and its pattern of distribution. Simplifying Equation (17) further, we know that:

$$\theta = \omega t$$

Therefore,

$$E = \frac{-t^2}{\sqrt{[\frac{T_r}{\mu W}]^2 + [\frac{\rho\omega t}{2\pi}]^2}} \qquad (18)$$

Therefore, $E = g(W, t)$. Note that T_r, μ, ω, ρ are constants depending on the nature of the materials of the workpiece. Now, the heat generated is calculated as:

$$Q = T_r \omega t \quad (19)$$

Therefore, the increase temperature ΔT is derived from:

$$MC_m \Delta T = T_r \omega t \quad (20)$$

That is:

$$\Delta T = \frac{T_r \omega t}{MC_m} \quad (21)$$

Thus, Equation (18) gives the value for the temperature distributed while Equation (21) shows how it is distributed. Now,

$$W = mg \quad (22)$$

Also,

$$m = \rho_m \pi r^2 L \quad (23)$$

Therefore:

$$W = \rho_m \pi r^2 L_g \quad (24)$$

But,

$$r = R - \alpha$$

Hence,

$$W = \rho_m \pi (R-\alpha)^2 L_g \quad (25)$$

Therefore, the weight of the workpiece depends on the depth cut. Putting Equation (25) into (18) we have:

$$E = \frac{-t^2}{\frac{1}{\pi}\sqrt{\frac{T_r^2}{\mu^2[\rho_m^2 (R-\alpha)^4 L^2 g^2]} + [\frac{\rho \omega t}{2}]^2}} \quad (26)$$

This is the same as:

$$E = \frac{-\pi t^2}{\sqrt{\frac{T_r^2}{\mu^2[\rho_m^2 (R-\alpha)^4 L^2 g^2]} + \frac{\rho^2 \omega^2 t^2}{4}}} \quad (27)$$

This model is not limited to cold and hot conditions but "red" hot machining conditions. For cold metal working conditions, the model proposed by Oke *et al.* (2006) could be slightly adjusted to incorporate calculations relevant to coolants. In normal practice, coolants (liquid) are employed to reduce heat due to the friction from cutting tools and the material being processed. However, for "red" hot condition, which is the focus of the current paper, it is not necessary to incorporate the effect of coolant on heat dissipations reduction since the temperature is significantly higher than that of coolant, and the coolant is of no effect.

3. Case study

In order to show the practical application of the mathematical relations just derived, which is based on the theory of complex applied potential, it will be necessary to give corresponding practical examples. Consider an experiment being performed in a machine laboratory on steel. The steel was turn-down through a depth of 0.001m for each turn. The initial diameter of the steel was 0.07m. If the length of the steel was 1m and the lathe machine was to run at 600rpm.

It is required to determine the temperature of the workpiece after the second turn. However, it should be noted that the operation was carried out for 30 minutes and it takes the operator 5 minutes to turn-down the materials once. The following are the conditions relevant to the steel used: ρ = 7850 kg/m^3, $\mu = 0.1$, g = 9.87 m/s^2, C_m = 460 kJ/kgk. The initial temperature of the steel was 20°C. It is required to establish a relationship among E vs t and C vs t. By choosing a time interval $(0 < t < 30)$ minutes, we determine if the curves were orthogonal or not? In solving this problem, we first calculate the mass of workpiece. Initially,

$$M_0 = \rho m \pi R^2 L = 7850 \times \pi \times \frac{0.070^2}{4} \times 1 = 30.21kg$$

It should be noted that after first turn,

$$r = \frac{0.07}{2} - 0.001 = 0.034\text{m}.$$

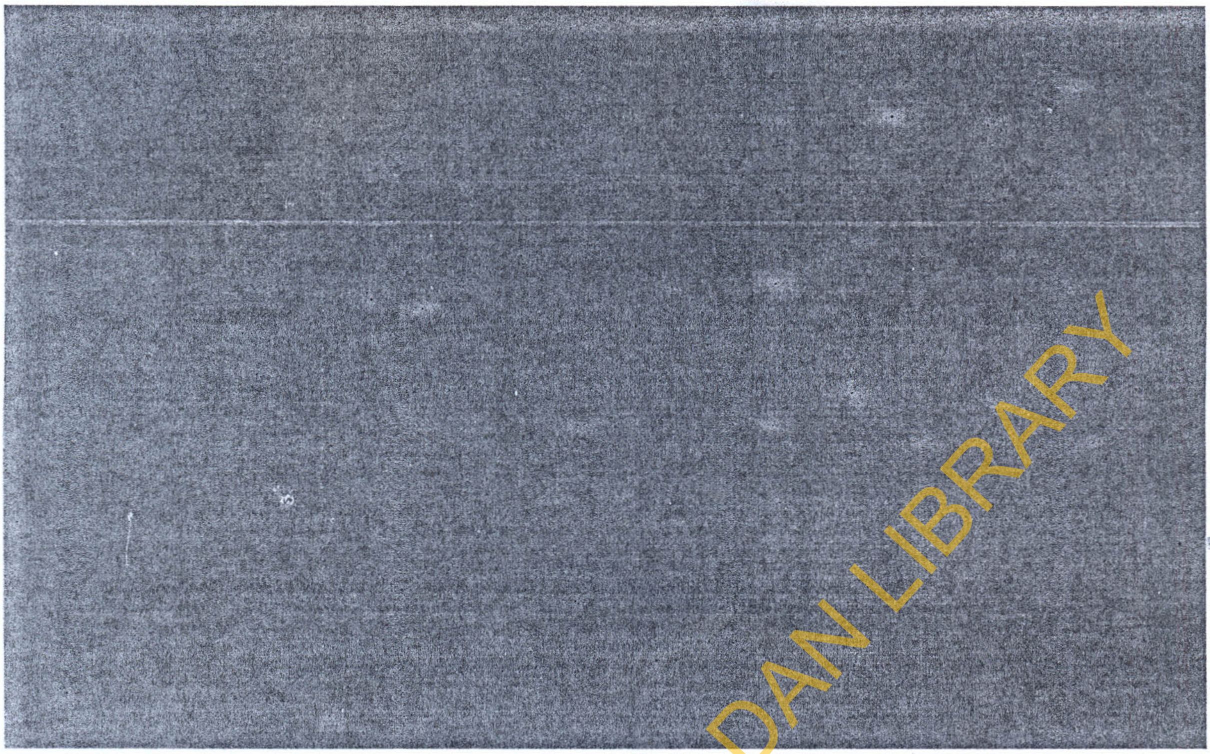

Figure 2. Sensitivity analysis of parameter (R) input in relation to parameter M - output with changes in machine operations parameter values.

Table 1. Monte Carlo sampling data on machining conditions (zinc).

Problem	D (m) Diameter	Depth of cut (m)	Length (m)	Speed (rpm)	Mass (Kg) M_o	Weight (N) W	Torque (Nm) T_r	Angular speed ($rads^{-1}$) ω	T_1+20°C	T_2
1	0.035	0.001	0.94	846	25.83	56.658	0.098	88.59	21.2	23.5
2	0.041	0.005	0.86	825	32.43	75.743	0.123	86.39	21.1	23.2
3	0.037	0.002	0.88	606	27.02	53.042	0.092	63.46	20.8	22.5
4	0.046	0.001	0.98	852	46.51	105.011	0.243	89.22	21.6	24.6
5	0.056	0.001	0.85	982	59.79	137.187	0.389	102.83	20.3	24.7
6	0.037	0.002	0.97	975	29.79	58.466	0.102	102.10	21.4	24.1
7	0.049	0.005	0.96	715	51.70	80.818	0.165	74.87	21.2	23.5
8	0.056	0.003	0.88	879	51.90	121.767	0.320	92.05	21.8	25.5
9	0.058	0.004	0.85	704	54.14	117.615	0.309	73.72	21.5	24.3
10	0.052	0.002	0.97	995	58.83	123.697	0.312	104.20	22.0	26.0
11	0.069	0.003	0.99	674	105.73	217.481	0.719	70.58	21.8	25.4
12	0.046	0.002	0.97	815	46.04	94.706	0.209	85.35	21.4	24.3
13	0.055	0.002	0.95	964	54.46	136.763	0.366	100.95	22.1	26.2
14	0.069	0.003	0.89	871	95.05	195.513	0.647	91.21	22.3	26.9
15	0.067	0.005	0.93	775	93.64	167.239	0.500	81.16	21.9	25.6
16	0.061	0.001	0.89	934	74.28	171.474	0.531	97.81	22.3	26.9
17	0.054	0.004	0.85	898	55.60	99.50	0.240	94.04	21.7	25.2
18	0.061	0.005	0.88	727	73.45	126.686	0.339	76.13	21.6	24.7
19	0.052	0.002	0.92	601	55.80	117.321	0.296	52.94	21.0	22.8
20	0.037	0.001	0.93	762	28.56	63.056	0.116	79.80	21.1	23.3

Table 2. Monte Carlo sampling data on machining conditions (aluminum).

Problem	D (m) Diameter	Depth of cut (m)	Length (m)	Speed (rpm)	Mass (Kg) M_o	Weight (N) W	Torque (Nm) T_r	Angular speed ($rads^{-1}$) ω	T_1+20°C	T_2
1	0.039	0.001	0.86	683	2.498	24.65	0.06	71.52	20.58	20.94
2	0.056	0.005	0.95	955	4.263	42.09	0.12	100.0	20.951	21.58
3	0.049	0.002	0.88	738	3.779	37.31	0.105	77.28	20.725	20.73
4	0.045	0.002	0.94	825	3.351	33.09	0.085	86.39	20.74	21.23
5	0.055	0.003	0.92	948	4.684	46.25	0.143	0.10	21.031	21.09
6	0.064	0.002	0.94	884	7.176	70.86	0.266	0.10	21.252	21.32
7	0.066	0.005	0.99	674	6.584	65.01	0.228	70.58	20.826	21.37
8	0.067	0.005	0.91	956	6.270	67.35	0.240	100.1	20.357	21.14
9	0.035	0.004	0.89	649	4.53	44.73	0.137	67.76	20.692	20.94
10	0.051	0.003	0.89	679	3.822	37.74	0.106	71.10	20.067	20.52
11	0.044	0.002	0.98	722	3.325	32.83	0.082	75.61	20.630	21.04
12	0.054	0.004	0.89	909	3.994	39.43	0.113	95.19	20.91	21.51
13	0.066	0.003	0.91	757	6.947	68.60	0.257	79.27	20.297	20.95
14	0.054	0.003	0.99	778	4.837	47.76	0.143	81.47	20.814	21.33
14	0.065	0.002	0.87	792	6.865	67.79	0.258	82.94	21.053	21.75
16	0.052	0.005	0.82	861	3.067	30.29	0.080	90.16	20.794	21.31
17	0.036	0.003	0.85	825	1.622	16.02	0.030	86.39	20.54	20.90
18	0.062	0.004	0.89	992	5.503	54.34	0.183	103.88	21.167	22.44
19	0.042	0.004	0.91	844	2.230	22.03	0.047	88.38	20.629	21.04
20	0.044	0.001	0.83	838	3.105	30.58	0.080	87.76	20.766	21.27

Table 3. Monte Carlo sampling data on machining conditions (steel).

Problem	D (m) Diameter	Depth of cut (m)	Length (m)	Speed (rpm)	Mass (Kg) M_o	Weight (N) W	Torque (Nm)T_r	Angular speed ($rads^{-1}$)ω	T_1+20°C	T_2
1	0.068	0.003	0.87	952	24.81	203.59	1.38	99.69	24.4	25.25
2	0.063	0.004	0.96	984	23.50	183.27	1.28	103.04	24.3	25.02
3	0.063	0.005	0.96	637	23.50	170.42	1.07	56.706	22.3	22.71
4	0.063	0.005	0.96	907	23.50	170.42	1.07	94.98	22.3	22.99
5	0.051	0.003	0.91	633	14.60	112.15	0.57	56.29	21.9	22.25
6	0.042	0.005	0.89	995	9.68	55.47	0.23	104.20	22.8	23.26
7	0.052	0.004	0.87	973	14.51	102.51	0.53	101.89	23.5	24.11
8	0.065	0.004	0.89	698	23.19	175.98	1.14	73.20	23.1	23.67
9	0.053	0.005	0.95	667	16.46	106.90	0.57	59.85	22.1	23.53
10	0.062	0.005	0.86	604	20.39	141.53	0.88	53.25	22.2	22.58
11	0.061	0.005	0.99	632	22.72	156.71	0.96	56.18	22.3	22.69
12	0.066	0.003	0.91	755	24.45	199.38	1.32	79.06	23.4	24.05
13	0.059	0.001	0.95	708	21.50	187.85	1.12	74.14	22.9	23.48
14	0.058	0.005	0.93	721	19.30	130.41	0.76	75.50	22.9	23.40
15	0.068	0.001	0.92	893	26.24	243.90	1.66	93.51	24.2	25.05
16	0.046	0.003	0.98	923	12.79	95.43	0.44	96.66	22.9	23.43
17	0.065	0.005	0.99	991	25.80	182.26	1.18	103.77	24.4	25.18
18	0.047	0.004	0.96	954	13.08	88.87	0.42	99.90	23.1	23.63
19	0.038	0.001	0.98	952	7.66	67.83	0.26	99.69	22.5	22.99
20	0.035	0.003	0.94	637	7.48	50.67	0.18	56.71	21.3	21.56

Table 4. Monte Carlo sampling data on machining conditions (copper).

Problem	D (m) Diameter	Depth of cut (m)	Length (m)	Speed (rpm)	Mass (Kg)M_o	Weight (N) W	Torque (Nm)T_r	Angular speed (rads^{-1}) ω	T_1+20°C	T_2
1	0.068	0.003	0.93	962	121.05	248.303	7.466	100.74	43.3	91.3
2	0.063	0.004	0.87	988	97.20	189.500	5.147	103.46	51.6	95.4
3	0.045	0.002	0.97	751	55.29	118.846	2.421	78.64	30.0	54.8
4	0.037	0.002	0.96	855	36.99	77.080	1.271	89.54	31.3	54.0
5	0.062	0.005	0.89	606	96.30	167.152	4.216	63.46	32.3	57.7
6	0.053	0.004	0.99	988	78.28	145.500	3.246	103.46	37.8	73.6
7	0.066	0.002	0.98	868	120.16	261.652	7.868	90.87	41.0	84.3
8	0.054	0.003	0.94	959	77.49	150.427	3.502	100.43	38.0	75.0
9	0.053	0.002	0.99	981	78.61	158.428	5.837	102.73	49.1	86.1
10	0.058	0.002	0.99	763	94.15	200.519	5.251	79.90	36.1	74.8
11	0.069	0.003	0.97	693	130.55	275.960	8.298	72.57	36.8	72.0
12	0.066	0.005	0.93	605	114.52	202.569	5.502	63.35	33.2	60.5
13	0.035	0.005	0.88	739	30.47	41.318	0.521	77.39	27.5	42.4
14	0.064	0.003	0.92	729	106.53	214.961	6.047	76.34	36.5	70.6
15	0.045	0.001	0.95	894	54.38	127.745	2.726	93.62	35.4	66.4
16	0.068	0.003	0.97	746	126.79	258.982	7.727	78.12	27.9	65.2
17	0.049	0.002	0.87	652	59.05	127.864	2.853	68.28	31.7	55.3
18	0.046	0.003	0.97	677	58.02	107.797	2.691	70.90	33.6	55.4
19	0.063	0.001	0.85	696	95.37	226.943	6.824	72.88	36.9	68.4
20	0.047	0.004	0.93	953	58.08	103.351	2.005	99.80	34.9	64.8

Table 5. Statistical analysis showing t-test results for significant tests temperature changes between materials made of steel and copper.

Problem	Steel	Copper	Statistical descriptions	Steel	Copper
1	25.25	91.30	Mean	23.5415	68.4
2	25.02	95.40	Variance	1.017392	187.4684
3	22.71	54.80	Observations	20	20
4	22.99	54.00	Correlation	0.343915	
5	22.25	57.70	Hypothesized Mean	0	
6	23.26	73.60	Df	19	
7	24.11	84.30	t Stat	-14.9951	
8	23.67	75.00	P(T<=t) one tail	2.77E-12	
9	23.53	86.10	t Critical one tail	1.729131	
10	22.58	74.80	P(T<=t) two tail	5.54E-12	
11	22.69	72.00	t Critical two tail	2.093025	
12	24.05	60.50			
13	23.48	42.40			
14	23.40	70.60			
15	25.05	66.40			
16	23.43	65.20			
17	25.18	55.30			
18	23.63	55.40			
19	22.99	68.40			
20	21.56	64.80	Decision	The differences are significant	

Table 6. Statistical analysis showing t-test results for significant tests temperature changes between materials made of copper and zinc.

Problem	Copper	Zinc	Statistical descriptions	Copper	Zinc
1	91.30	23.5	Mean	68.4	24.66
2	95.40	23.2	Variance	187.4684	1.684632
3	54.80	22.5	Observations	20	20
4	54.00	24.6	Correlation	-0.26989	
5	57.70	24.7	Hypothesized Mean	0	
6	73.60	24.1	Df	19	
7	84.30	23.5	t Stat	13.87538	
8	75.00	25.5	P(T<+t) one tail	1.08E-11	
9	86.10	24.3	t Critical one tail	1.729131	
10	74.80	26.0	P(T<+t) two tail	2.15E-11	
11	72.00	25.4	t Critical two tail	2.093025	
12	60.50	24.3			
13	42.40	26.2			
14	70.60	26.9			
15	66.40	25.6			
16	65.20	26.9			
17	55.30	25.2			
18	55.40	24.7			
19	68.40	22.8			
20	64.80	23.3	Decision	The differences are significant	

Table 7. Statistical analysis showing t-test results for significant tests temperature changes between material of made up steel and aluminum.

Problem	Steel	Aluminum	Statistical descriptions	Steel	Aluminum
1	25.25	20.94	Mean	23.5415	21.22
2	25.02	21.58	Variance	1.017392	0.167821
3	22.71	20.73	Observations	20	20
4	22.99	21.23	Correlation	0.257536	
5	22.25	21.09	Hypothesized Mean	0	
6	23.26	21.32	Df	19	
7	24.11	21.37	t Stat	10.52847	
8	23.67	21.14	P(T<+t) one tail	1.14E-09	
9	23.53	20.94	t Critical one tail	1.729131	
10	22.58	20.52	P(T<+t) two tail	2.28E-09	
11	22.69	21.04	t Critical two tail	2.093025	
12	24.05	21.51			
13	23.48	20.95			
14	23.40	21.33			
15	25.05	21.25			
16	23.43	21.31			
17	25.18	20.90			
18	23.63	22.44			
19	22.99	21.04			
20	21.56	21.27	Decision	The differences are not significant	

Table 8. Statistical analysis showing t-test results for significant tests of temperature changes between materials made up of steel and zinc.

			Statistical descriptions		
Problem	**Steel**	**Zinc**		**Steel**	**Zinc**
1	25.25	23.5	Mean	23.5415	24.66
2	25.02	23.2	Variance	1.017392	1.684632
3	22.71	22.5	Observations	20	20
4	22.99	24.6	Correlation	0.020551	
5	22.25	24.7	Hypothesized Mean	0	
6	23.26	24.1	Df	19	
7	24.11	23.5	t Stat	-3.07379	
8	23.67	25.5	P(T<=t) one tail	0.003125	
9	23.53	24.3	t Critical one tail	1.729131	
10	22.58	26.0	P(T<=t) two tail	0.006249	
11	22.69	25.4	t Critical two tail	2.093025	
12	24.05	24.3			
13	23.48	26.2			
14	23.40	26.9			
15	25.05	25.6			
16	23.43	26.9			
17	25.18	25.2			
18	23.63	24.7			
19	22.99	22.8			
20	21.56	23.3	Decision	The differences are not significant	

Table 9. Statistical analysis showing t-test results for significant tests of temperature changes between materials made of copper and aluminum.

			Statistical descriptions		
Problem	**Copper**	**Aluminum**		**Copper**	**Aluminum**
1	91.30	20.94	Mean	68.4	21.22
2	95.40	21.58	Variance	187.4684	0.167821
3	54.80	20.73	Observations	20	20
4	54.00	21.23	Correlation	-0.04422	
5	57.70	21.09	Hypothesized Mean	0	
6	73.60	21.32	Df	19	
7	84.30	21.37	t Stat	15.38301	
8	75.00	21.14	P(T<=t) one tail	1.76E-12	
9	86.10	20.94	t Critical one tail	1.729131	
10	74.80	20.52	P(T<=t) two tail	3.53E-12	
11	72.00	21.04	t Critical two tail	2.093025	
12	60.50	21.51			
13	42.40	20.95			
14	70.60	21.33			
15	66.40	21.25			
16	65.20	21.31			
17	55.30	20.90			
18	55.40	22.44			
19	68.40	21.04			
20	64.80	21.27	Decision	The differences are significant	

The weight is calculated as follows:

$$W = \pi r^2 L \times \rho m \times g$$

$$= \pi \times 0.034^2 \times 1 \times 7850 \times 9.87 = 281.138\ N$$

The torque is calculate as follows:

$$T_r = \mu W r = 0.1 \times 281.138 \times 0.034 = 0.96\ Nm$$

Now, we calculate the angular speed as follows:

$$\omega = 600 \times \frac{2\pi}{60} = 62.83\ rads^{-1}$$

Note that time taken for the first turn

$$t = 5 \times 60 = 300\ s$$

Therefore the temperature increase is calculated as follows:

$$\Delta T = \frac{60.83 \times 300 \times 0.96}{(281.38/9.87) \times 460} = 1.4^oC$$

Again, the temperature after turn $T_1 = 21.4^oC$. After second turn change in temperature:

$$\Delta T = wt \times \mu \times r_2 g \times \frac{1}{C_m} =$$

$$\frac{62.83 \times 300 \times 0.1 \times 9.87 \times (0.035-0.002)}{460} = 1.3^oC$$

Note that the temperature after second turn $T_2 = 21.4^oC + 1.3^o = 22.7^oC$. The results, with details shown in the appendix, suggest that the relationship among E and t, and C and t are orthogonal. Part of the solution to the problem discussed here entails developing a computer code for the model developed and running it on Matlab computer program.

The results of the variations of parameters are shown in the appendix. Program verification and accuracy determination is done through extensive program testing, which has been completed in this work. However, further extension of program and subsequent refinements and testing is encouraged. Monte Carlo sampling is used as a technique to increase the size of the experimental sample considered in the case study section for the purpose of increasing the confidence of decisions taken. It has been widely applied in empirical and theoretical studies and is claimed to be a very effective tool in testing newly developed models in productions systems (Nahmias, 2001).

Apart from steel, three other materials were tested: aluminum, copper, and zinc with densities of 2700kgm^{-3}, 960kgm^{-3}, and 7140kgm^{-3} respectively. Only g and Cm were held constant while other parameters were varied. It is important to note that μ was taken as 0.125, 0.970, and 0.105, for aluminum, copper, and zinc, respectively. For the purpose of Monte Carlo sampling, additional information relating to the ranges of values selected are as follows: Radius ranges between 0.035m and 0.070m, depth of cut between 0.001and 0.005m, length between 0.85 and 1.00 m, and the speed between 600rpm. Furthermore, the heat capacities of copper, zinc and aluminum are 385 J/Kg.k, 388 J/Kg.k and 888 J/Kg.k, respectively. The tables displaying the results are shown in Tables 1,2,3 and 4. Also displayed in Tables 5 to 9 are the results of the t-test carried out. The results of sensitivities test are displayed in Figure 2. From the results obtained, it is found that no significant differences exist in the model behavior of steel and aluminum, steel and zinc compared to other different work piece materials.

The explanation for this is as follows. Steel has a very high specific heat capacity that far exceeds that of aluminum, zinc and copper. As such, steel would require a very high temperature to melt when aluminum, zinc and copper would have melted. Thus, if it is reported that no significant differences exist between steel and aluminum at "red hot" temperature, it means that the recrystalisation temperature of aluminum is not reached to cause structural changes of the materials of steel and aluminum. This same argument explains why no significant differences may exist between steel and zinc.

4. Conclusion

The current paper has presented a mathematical model that tracks temperature distribution during both cold and hot working machining conditions. The import aspect of the model is that it could be used to detect what relationship exists among E and t, C and t, and suggest if it is orthogonal or not. With this, plots of graphs could be established such that it would be easy to determine at what temperature materials would demonstrate deviant behavior and be able to compare behavior of two or more materials at different temperature. In "red-hot' conditions, plastic deformation of the workpiece may take place.

Since this may not be desired in many products the mechanical properties of the material, which may be altered, thus making the final product unacceptable, particularly if required for moving parts of machineries that may be subjected to high torsional forces, the material may break. This model can be used for milling and surfacing activities in high temperature applications. Considering the management implication, it is noted that if knowledge of temperature distribution in materials were known, the operator would achieve high performance and results in machining. The operator could advise customers and management on the appropriate material to utilize for component manufacture. Due to the difficulties that may exist in machining activities, poor knowledge on temperature distribution in materials during machining would make machine operators unwilling to accept responsibilities. In addition, even though machining a material for component manufacture may demand much human efforts and energy, the machining operator would be enthusiastic and determined to succeed if he has knowledge of temperature in materials during machining.

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Appendix

Code

```
% A program to simulate the influence of coef-
ficient of friction, u
% on the path of temperature distribution, E

% Assuming the following constants
p = 7850;   % density of workpiece in kg/cubic
meter
r = 0.05;   % radius of cylinder cut in meters
pm = 0.02; % pitch in meters
R = 0.07; % initial radius of bar in meters
al = 0.001;% cutting depth in meters
L = 5;  % length of bar in meters
g = 9.87;   %  acceleration due to gravity
w  =  10;  %  angular  velocity  in  radians  per
second
t = 300; % time in seconds
Tr = 1200; % torque on material

fprintf('%s\t','coefficient   of   friction','
   temperature distribution')
fprintf('%s\n',' ')
for u = 0:+0.02:0.2    % Range of coefficient of
friction
   %for neat presentation of results
   E                    =                        -
pi*(t^2)/sqrt((Tr^2)/((u^2)*(((pm*L*g)^2)*((R-
r)^4)))+((p*w*t)^2)/4);
              %  calculation  of  the  path  of
temperature distribution
           fprintf('%18.2f\t',u,E)
         fprintf('%s\n',' ')  % results
      end
 end
```

Results

» coefficient of friction	temperature distribution
0.00	0.00
0.02	-0.00
0.04	-0.00
0.06	-0.01
0.08	-0.01
0.10	-0.01
0.12	-0.01
0.14	-0.01
0.16	-0.01
0.18	-0.01
0.20	-0.01

Code

```
% A program to simulate the influence of time,
t
% on the path of temperature distribution, E

% Assuming the following constants
p = 7850;  % density of workpiece in kg/cubic
meter
r = 0.05;  % radius of cylinder cut in meters
pm = 0.02; % pitch in meters
R = 0.07; % initial radius of bar in meters
al = 0.001; % cutting depth in meters
L = 5;  % length of bar in meters
g = 9.87;  %  acceleration due to gravity
w = 10;  % angular velocity in radians per
second
u = 0.1; % coefficient of friction
Tr = 1200; % torque on material

fprintf('%s\t','Time (in seconds)',' tempera-
ture distribution')
fprintf('%s\n',' ')
for t = 300:+100:1800 %  Range of time in
seconds
   %for neat presentation of results
   E                        =                 -
pi*(t^2)/sqrt((Tr^2)/((u^2)*(((pm*L*g)^2)*((R-
r)^4)))+((p*w*t)^2)/4);
                  % calculation of the path of
temperature distribution
           fprintf('%18.2f\t',t,E)
        fprintf('%s\n',' ')  % results
     end
end
```

Results

» Time (in seconds)	temperature distribution
300.00	-0.01
400.00	-0.01
500.00	-0.02
600.00	-0.03
700.00	-0.04

» Time (in seconds)	temperature distribution
800.00	-0.05
900.00	-0.05
1000.00	-0.06
1100.00	-0.07
1200.00	-0.08
1300.00	-0.09
1400.00	-0.10
1500.00	-0.11
1600.00	-0.12
1700.00	-0.12
1800.00	-0.13

Code

```
% A program to simulate the influence of ra-
dius, r
% on the path of temperature distribution, E

% Assuming the following constants
p = 7850;  % density of workpiece in kg/cubic
meter
t = 300; % time in seconds
pm = 0.02; % pitch in meters
R = 0.07; % initial radius of bar in meters
al = 0.001; % cutting depth in meters
L = 5;  % length of bar in meters
g = 9.87;  %  acceleration due to gravity
w = 10;  % angular velocity in radians per
second
u = 0.1; % coefficient of friction
Tr = 1200; % torque on material

fprintf('%s\t','Radius (in meters)',' tempera-
ture distribution')
fprintf('%s\n',' ')
for r = 0.005:+0.005:0.065% Range of radius in
meters
   %for neat presentation of results
   E                        =                 -
pi*(t^2)/sqrt((Tr^2)/((u^2)*(((pm*L*g)^2)*((R-
r)^4)))+((p*w*t)^2)/4);
                  % calculation of the path of
temperature distribution
           fprintf('%18.2f\t',r,E)
        fprintf('%s\n',' ')  % results
     end
end
```

Results

» Radius (in meters)	temperature distribution
0.01	-0.02
0.01	-0.02
0.01	-0.02
0.02	-0.02
0.03	-0.02

» Radius (in meters)	temperature distribution
0.03	-0.02
0.03	-0.02
0.04	-0.02
0.04	-0.01
0.05	-0.01
0.06	-0.01
0.06	-0.00
0.07	-0.00

Code

```
% A program to simulate the influence of
length, L
% on the path of temperature distribution, E

% Assuming the following constants
p = 7850;  % density of workpiece in kg/cubic
meter
t = 300; % time in seconds
pm = 0.02; % pitch in meters
r = 0.05;  % radius of cylinder cut in meters
al = 0.001;% cutting depth in meters
R = 0.07; % initial radius of bar in meters
g = 9.87;  %  acceleration due to gravity
w  =  10;  %  angular velocity in radians per
second
u = 0.1; % coefficient of friction
Tr = 1200; % torque on material
fprintf('%s\t','Length (in meters)',' tempera-
ture distribution')
fprintf('%s\n',' ')
for L = 1:20   % Range of radius in meters
   %for neat presentation of results
   E                                  =                         -
pi*(t^2)/sqrt((Tr^2)/((u^2)*(((pm*L*g)^2)*((R-
r)^4)))+((p*w*t)^2)/4);
                 % calculation of the path of
temperature distribution
           fprintf('%18.2f\t',L,E)
         fprintf('%s\n',' ')  % results
      end
 end
```

Results

» Length (in meters)	temperature distribution
1.00	-0.00
2.00	-0.00
3.00	-0.01
4.00	-0.01
5.00	-0.01
6.00	-0.01
7.00	-0.01
8.00	-0.01
9.00	-0.01
10.00	-0.01
11.00	-0.02

» Length (in meters)	temperature distribution
12.00	-0.02
13.00	-0.02
14.00	-0.02
15.00	-0.02
16.00	-0.02
17.00	-0.02
18.00	-0.02
19.00	-0.02
20.00	-0.02

Code

```
% A program to simulate the influence of ra-
dius, R
% on the path of temperature distribution, E

% Assuming the following constants
p = 7850;  % density of workpiece in kg/cubic
meter
t = 300; % time in seconds
pm = 0.02; % pitch in meters
r = 0.05;  % radius of cylinder cut in meters
al = 0.001; % cutting depth in meters
L = 5;  % length of bar in meters
g = 9.87;  %  acceleration due to gravity
w  =  10;  %  angular velocity in radians per
second
u = 0.1; % coefficient of friction
Tr = 1200; % torque on material
fprintf('%s\t','Radius (in meters)',' tempera-
ture distribution')
fprintf('%s\n',' ')
for R = 0.055:+0.005:0.10 % Range of radius in
meters
   %for neat presentation of results
   E                                  =                         -
pi*(t^2)/sqrt((Tr^2)/((u^2)*(((pm*L*g)^2)*((R-
r)^4)))+((p*w*t)^2)/4);
                 % calculation of the path of
temperature distribution
           fprintf('%18.2f\t',R,E)
         fprintf('%s\n',' ')  % results
      end
 end
```

Results

» Radius (in meters)	temperature distribution
0.06	-0.00
0.06	-0.00
0.07	-0.01
0.07	-0.01
0.07	-0.01
0.08	-0.02
0.08	-0.02
0.09	-0.02
0.10	-0.02
0.10	-0.02

Code

```
% A program to simulate the influence of angu-
lar velocity, w
% on the path of temperature distribution, E

% Assuming the following constants
p = 7850;   % density of workpiece in kg/cubic
meter
t = 300; % time in seconds
pm = 0.02; % pitch in meters
r = 0.05;   % radius of cylinder cut in meters
al = 0.001; % cutting depth in meters
R = 0.07; % initial radius of bar in meters
g = 9.87;   %   acceleration due to gravity
L = 5;  % length of bar in meters
u = 0.1; % coefficient of friction
Tr = 1200; % torque on material
fprintf('%s\t','Angular   velocity   (in   ra-
dians/sec)',' temperature distribution')
fprintf('%s\n',' ')
for w = 1:20    % Range of angular velocity in
radians per second
   %for neat presentation of results
   E                    =                      -
pi*(t^2)/sqrt((Tr^2)/((u^2)*(((pm*L*g)^2)*((R-
r)^4)))+((p*w*t)^2)/4);
                % calculation of the path of
temperature distribution
            fprintf('%24.2f\t',w,E)
          fprintf('%s\n',' ')  % results
      end
 end
```

Results

» Angular velocity (in radians/sec) temperature distribution

1.00	-0.01
2.00	-0.01
3.00	-0.01
4.00	-0.01
5.00	-0.01
6.00	-0.01
7.00	-0.01
8.00	-0.01
9.00	-0.01
10.00	-0.01
11.00	-0.01
12.00	-0.01
13.00	-0.01
14.00	-0.01
15.00	-0.01
16.00	-0.01
17.00	-0.01
18.00	-0.01
19.00	-0.01
20.00	-0.01