ESTIMATORS OF LINEAR REGRESSION MODEL WITH AUTOCORRELATED ERRROR TERMS AND PREDICTION USING CORRELATED UNIFORM REGRESSORS

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Abstract:

Performances of estimators of linear regression model with autocorrelated error term have been attributed to the nature and specification of the explanatory variables. The violation of assumption of the independence of the explanatory variables is not uncommon especially in business, economic and social sciences, leading to the development of many estimators. Moreover, prediction is one of the main essences of regression analysis. This work, therefore, attempts to examine the parameter estimates of the Ordinary Least Square estimator (OLS), Cochrane-Orcutt estimator (COR), Maximum Likelihood estimator (ML) and the estimators based on Principal Component analysis (PC) in prediction of linear regression model with autocorrelated error terms under the violations of assumption of independent regressors (multicollinearity) using Monte-Carlo experiment approach. With uniform variables as regressors, it further identifies the best estimator that can be used for prediction purpose by averaging the adjusted co-efficient of determination of each estimator over the number of trials.

Results reveal that the performances of COR and ML estimators at each level of multicollinearity over the levels of autocorrelation are convex – like while that of the OLS and PC estimators are concave; and that as the level of multicollinearity increases, the estimators perform much better at all the levels of autocorrelation. Except when the sample size is small (n=10), the performances of the COR and ML estimators are generally best and asymptotically the same. When the sample size is small, the COR estimator is still best except when the autocorrelation level is low. At these instances, the PC estimator is either best or competes with the best estimator. Moreover, at low level of autocorrelation in all the sample sizes, the OLS estimator competes with the best estimator in all the levels of multicollinearity.

Keywords: Prediction; Estimators; Autocorrelated error term;Multicollinearity.

1. Introduction

When the assumption of independence of error terms of the Classical Linear Regression Model is violated, as it is often found in time series data, the problem of autocorrelated error terms does not only arise but also results into Generalized Least Squares (GLS) Model. Aitken (1935) has shown that the GLS estimator β of β given as $\beta = (X^1\Omega^1X)^{-1} X^1\Omega^1Y$ is efficient among the class of linear unbiased estimators of β with variance – covariance matrix of β given as $V(\beta) = \sigma^2(X^1\Omega^1X)^{-1}$ where Ω is assumed to be known. However, Ω is not always known, it is often estimated by $\hat{\Omega}$ to have what is known as Feasible GLS estimator.

Several authors have worked on this violation especially in terms of the parameter estimation of the linear regression model when the error term follows autoregressive of orders one. The OLS estimator is inefficient even though unbiased. Its predicted values are also inefficient and the sampling variances of the autocorrelated error terms are known to be underestimated causing the t and the F tests to be invalid (Johnston, 1984; Fomby et al., 1984; Chartterjee, 2000; Maddala, 2002). To compensate for the lost of efficiency, several feasible GLS estimators have been developed. These include the estimator provided by Cochrane and Orcutt (1949), Paris and Winstern (1954), Hildreth and Lu (1960), Durbin (1960), Theil (1971), the maximum likelihood and the maximum likelihood grid (Beach and Mackinnon, 1978), and Thornton (1982). Chipman (1979), Kramer (1980), Kleiber (2001), Iyaniwura and Nwabueze (2004), Nwabueze (2005a, b, c), Ayinde and Ipinyomi (2007) and many other authors have not only observed the asymptotic equivalence of these estimators but have also noted that that their performances and efficiency depend on the structure of the regressor used. Rao and Griliches (1969) did one of the earliest Monte-Carlo investigations on the small sample properties of several two-stage regression methods in the context of autocorrelation error. Other recent works done on these estimators and the violations of the assumptions of classical linear regression model include that of Iyaniwura and Olaomi (2006), Ayinde and Oyejola (2007), Ayinde (2007a,b), Ayinde and Olaomi (2008), Ayinde (2008), and Ayinde and Iyaniwura (2008).

The violation of the assumption of independence explanatory variables leads to the problem of multcollinearity. This is often common in business and economics data. For instance, the independent variables such as family income and assets or store sales and number of employees or age and years of experience would tend to be highly correlated. With strongly interrelated regressors, interpretation given to the regression coefficients may no longer be valid because the assumption under which the regression model is built has been violated. Although the estimates of the regression coefficients provided by the OLS estimator is still unbiased as long as multicollinearity is not perfect, the regression coefficients may have large sampling errors which affect both the inference and forecasting that is based on the model (Chartterjee et al., 2000). Various methods have been developed to estimate the model parameters when muitcollinearity is present in a data set. These estimators include Ridge Regression estimator developed by Hoerl (1962) and Hoerl and Kennard (1970), Estimator based on Principal Component Regression suggested by Massy (1965), Marquardt (1970) and Bock, Yancey and Judge (1973), Naes and Marten (1988), and method of Partial Least Squares developed by Hermon Wold in the 1960s (Helland, 1988, Helland, 1990, Phatak and Jony 1997).

In spite of these several works done under these two assumption violations, no study has actually involved the two sets of estimators together; and since prediction is one of the basic reasons for regression analysis, this paper does examine the predictive potential of some of these estimators when the explanatory variables are correlated and uniformly distributed as most economic variables are always positive.

2. Materials and Methods

Consider the linear regression model of the form:

$$y_t = \beta_0 + \beta_1 x_{1t} + \beta_2 x_{2t} + \beta_3 x_{3t} + u_t \qquad (1)$$

Where $u_t = \rho u_{t-1} + \varepsilon_t$, $\varepsilon_t \sim N(0, \sigma^2)$, t = 1, 2, 3,...n and $X_i \sim U(0,1)$ i = 1, 2, 3 are correlated.

For Monte-Carlo simulation study, the parameters of equation (1) were specified and fixed as $\beta_0 = 4$, $\beta_1 = 2.5$, $\beta_2 = 1.8$ and $\beta_3 = 0.6$. The levels of multicollinearity among the independent variables were sixteen (16) and specified as: $\lambda(x_{12}) = \lambda(x_{13}) = \lambda(x_{23}) = -0.49, -0.4, -0.3, ..., 0.8, 0.9, 0.99.$ The levels of autocorrelation is twenty-one (21) and are specified as: $\rho = -0.99, -0.9, -0.8, ..., 0.8, 0.9, 0.99.$ Furthermore, the experiment was replicated in 1000 times (R =1000) under Six (6) levels of sample sizes (n =10, 15, 20, 30, 50, 100).

The correlated uniform regressors were generated by first using the equations provided by Ayinde (2007) and Ayinde and Adegboye (2010) to generate normally distributed random variables with specified intercorrelation. With P=3 the equations give:

$$X_1 = \mu_1 + \sigma_1 Z_1$$

$$X_2 = \mu_2 + \rho_{12}\,\sigma_2 Z_1 + \sqrt{m_{22}}\,Z_2 \qquad (2)$$

$$X_3 = \mu_3 + \rho_{13}\,\sigma_3 Z_1 + \frac{m_{23}}{\sqrt{m_{22}}} Z_2 + \sqrt{n_{33}}\,Z_3$$

Where $m_{22} = \sigma_2^2(1-\rho_{12}^2)$, $m_{23} = \sigma_2\sigma_3(\rho_{23} - \rho_{12}\rho_{13})$ and $n_{33} = m_{33} - \frac{m_{23}^2}{m_{22}}$; and $Z_i \sim N(0, 1)$ i = 1, 2, 3.

(The inter-correlation matrix has to be positive definite and hence, the correlations among the independent variable were taken as prescribed earlier). In the study, we assumed $X_i \sim N(0, 1)$, i = 1, 2, 3. We further utilized the properties of random variables that cumulative distribution function of Normal distribution produces U (0, 1) without affecting the correlation among the variables (Schumann, 2009) to generate $X_i \sim N(0,1), i = 1,2,3$.

The error terms were generated using one of the distributional properties of the autocorrelated error terms (u_t~N (0, $\frac{\sigma_\varepsilon^2}{1-\rho^2}$)) and the AR(1) equation as follows:

$$u_1=\frac{\varepsilon_1}{\sqrt{1-\rho^2}} \quad (3)$$

$$u_t = \rho u_{t-1} + \varepsilon_t \qquad t = 2,3,4,\ldots n \quad (4)$$

Since some of these estimators have now been incorporated into the Time Series Processor (TSP 5.0, 2005) software, a computer program was written using the software to estimate the Adjusted Coefficient of Determination of the model $(\bar{R}^2)$ the Ordinary Least Square (OLS) estimator, Cochrane Orcutt (COR) estimator, Maximum Likelihood estimator and the estimator based on Principal Component Analysis (PRN).

The Adjusted Coefficient of Determination of the model was averaged over the numbers of replications. i.e.

$$\bar{\bar{R}} = \frac{1}{R}\sum_{i=1}^{R} \bar{R}_i^2 \quad (5)$$

The two possible PCs (PC1 and PC2) of the Principal Component Analysis were used. Each provides its separate Adjusted Coefficient of Determination. An estimator is best if its Adjusted Coefficient of Determination is closest to unity.

3. Results and Discussion

The full summary of the simulated results of each estimator at different level of sample size, muticollinearity, and autocorrelation is contained in the work of Adebayo (2011). The graphical representations of the results when n=10, 15, 20, 30, 50 and 100 are respectively presented in Figure 1, 2, 3, 4, 5 and 6.

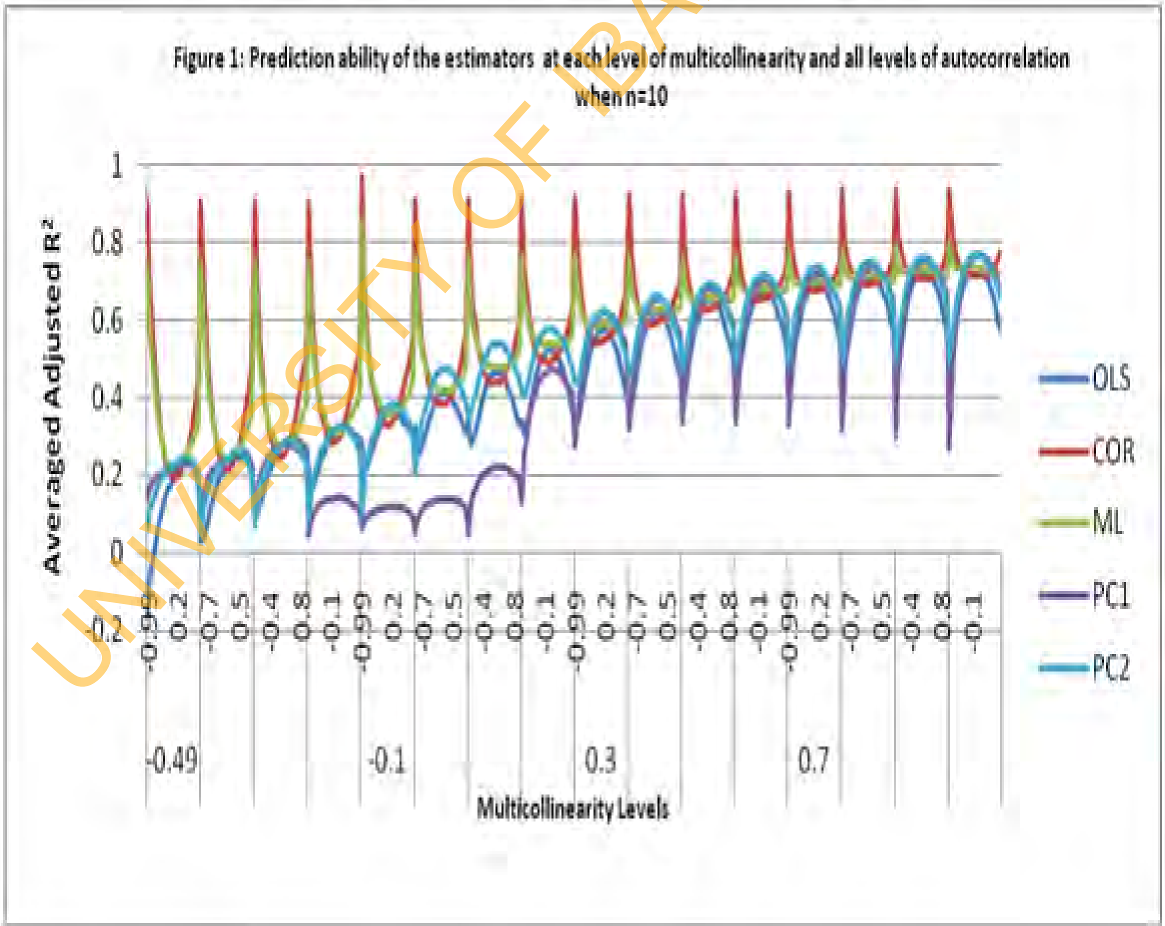


Figure 1: Prediction ability of the estimators at each level of multicollinearity and all levels of autocorrelation when n=10

Figure 2: Prediction ability of the estimators at each level of multicollinearity and all levels of autocorrelation when n=15

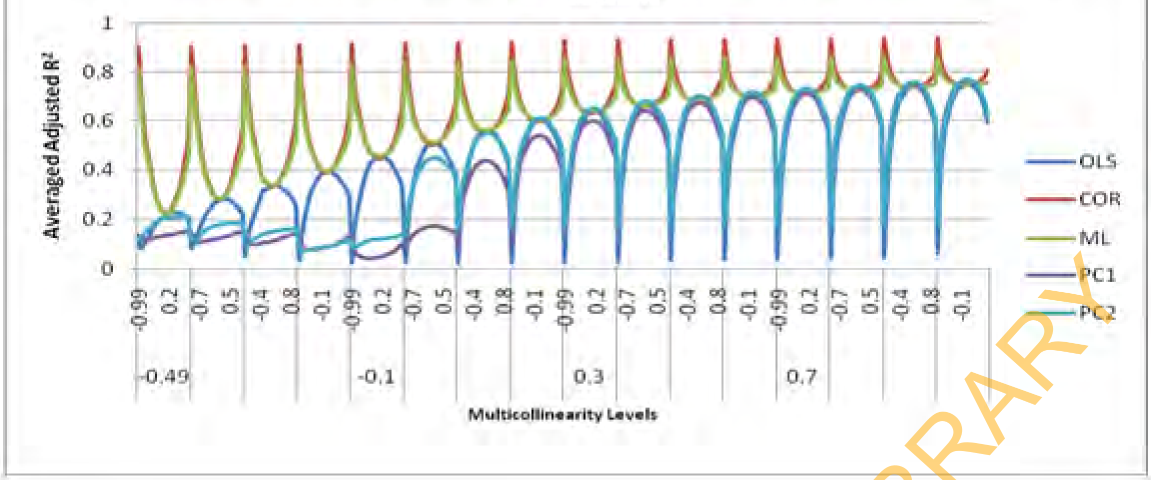


Figure 3: Prediction ability of the estimators at each level of multicollinearity and all levels of autocorrelation when n=20

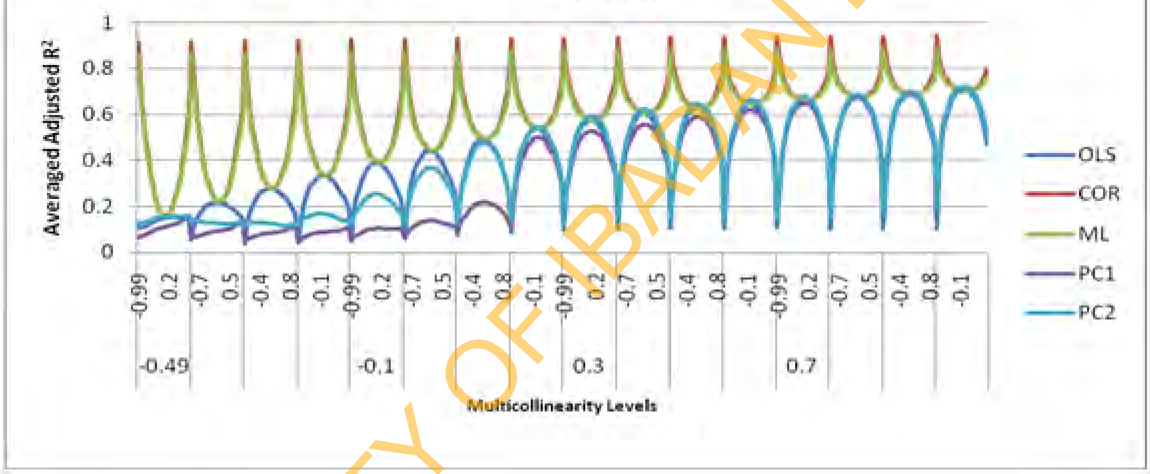
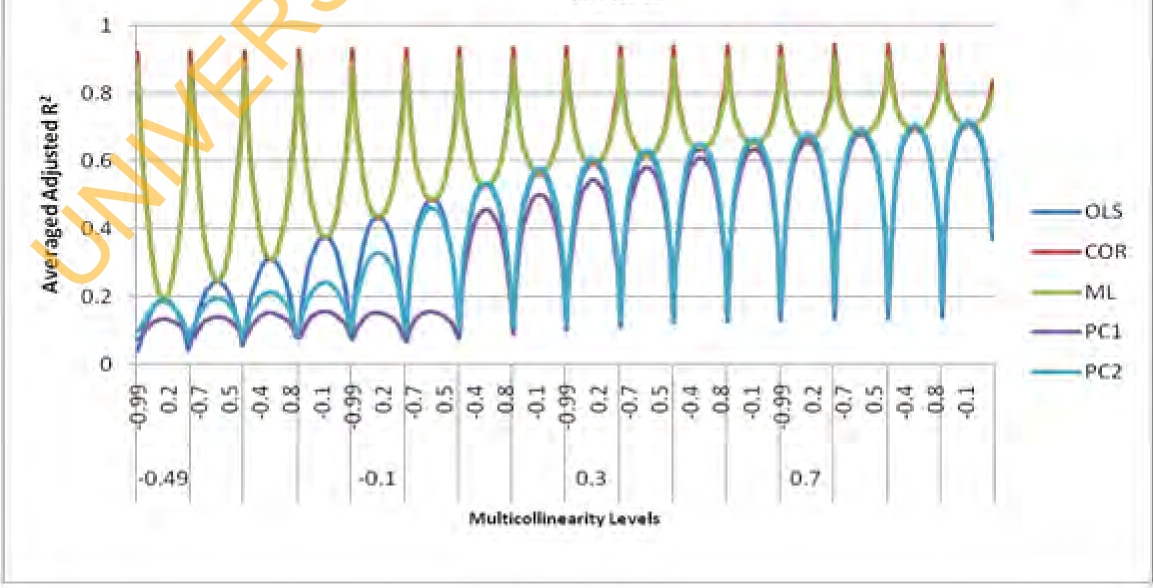


Figure 4: Prediction ability of the estimators at each level of multicollinearity and all levels of autocorrelation when n=30



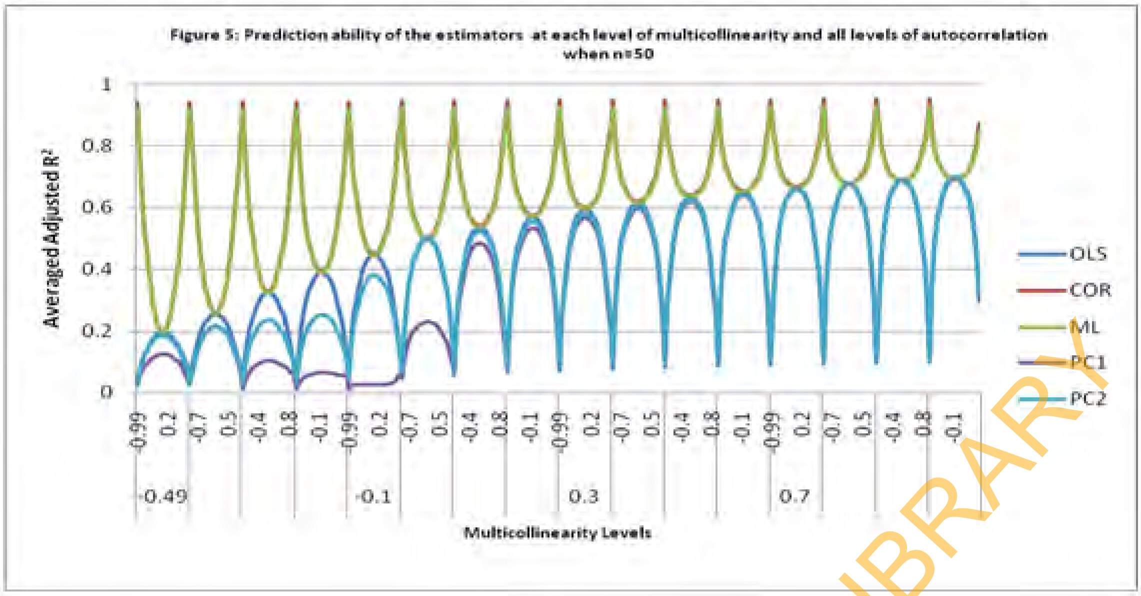


Figure 5: Prediction ability of the estimators at each level of multicollinearity and all levels of autocorrelation when n=50

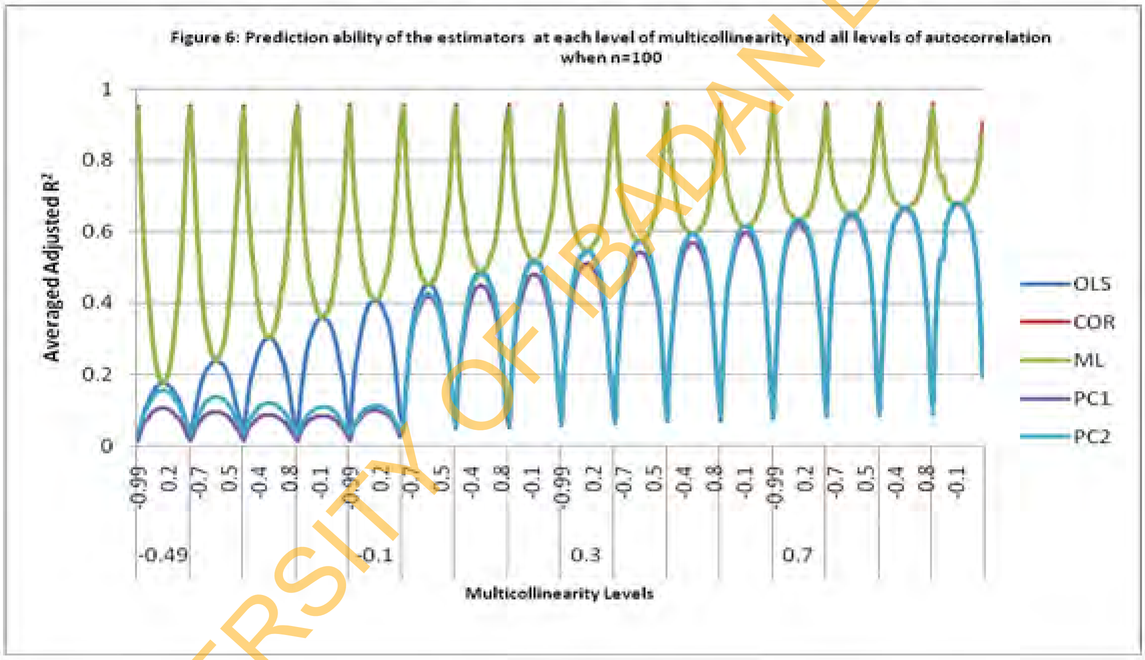


Figure 6: Prediction ability of the estimators at each level of multicollinearity and all levels of autocorrelation when n=100

These figures reveal that the performances of COR and ML estimators at each level of multicollinearity over the levels of autocorrelation are convex – like while that of the OLS and PC estimators are concave. Also, as the level of multicollinearity increases the estimators perform much better at all the levels of autocorrelation. The COR and ML estimators have high average adjusted coefficient of determination at all the levels of autocorrelation especially when $\lambda \geq 0.2$. At other instances, the values of COR and ML are only high at high levels of autocorrelation. Except when the sample size is small (n=10), the performances of the COR and ML estimators are generally best and asymptotically the same. However at low level of autocorrelation, the PC (PC2) estimator either performs better than or competes with the best estimator when $\lambda \leq -0.49$ and $\lambda \geq 0.1$. When the sample size is small (n =10), the COR estimator is best except when the autocorrelation level is low. At these instances, the PC2 estimator is either best or competes with the best estimator. Moreover, at low level of autocorrelation in all the sample sizes, the OLS estimator competes with the best estimator in all the levels of multicollinearity.

Very specifically in term of identification of the best estimator, Table 1, 2, 3, 4, 5 and 6 respectively summarize the best estimator for prediction at all the levels of autocorrelation and multicollinearity when the sample size is 10, 15, 20, 30, 50, 100.

From Table 1 when n =10, the best estimator is COR when $|\rho| \geq 0.7$ at all the levels of multicollinearity. Also, when $-0.3 \leq \rho \leq 0.6$ and $\lambda \geq 0$, and when $0 \leq \rho \leq 0.3$ and $\lambda \leq 0$ the estimators based on using PC2, and very sparsely PC1, are best. At other instances, the best estimator is generally ML and sparsely COR.

Table 1: The Best Estimator for Prediction at different level of Multicollinearity and Autocorrelation when n=10.

ρ	λ															
	-0.49	**-0.4**	**-0.3**	**-0.2**	**-0.1**	**0**	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**	**0.99**
-0.99	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.9	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.8	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.7	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.6	COR	COR	COR	COR	ML	ML	ML	ML	ML	ML	COR	COR	COR	COR	COR	COR
-0.5	COR	COR	ML	ML	ML	ML	ML	ML	ML	ML	ML	ML	ML	ML	ML	ML
-0.4	ML	ML	ML	ML	ML	ML	ML	ML	ML	ML	PR2	PR2	PR2	PR2	PR2	ML
-0.3	ML	ML	ML	ML	ML	PR2	PR2	PR2	PR2	PR2	PR2	PR2	PR2	PR2	PR2	PR2
-0.2	ML	ML	ML	ML	ML	PR2	PR2	PR2	PR2	PR2	PR2	PR2	PR2	PR2	PR2	PR2
-0.1	ML	ML	ML	ML	PR2	PR2	PR2	PR2	PR2	PR2	PR2	PR2	PR2	PR2	PR2	PR2
0	PR2	ML	PR1	PR2	PR2	PR2	PR2	PR2	PR2	PR2	PR2	PR2	PR2	PR2	PR2	PR2
0.1	PR2	PR2	PR2	PR2	PR2	PR2	PR2	PR2	PR2	PR2	PR2	PR2	PR2	PR2	PR2	PR2
0.2	PR2	PR2	PR2	PR2	PR2	PR2	PR2	PR2	PR2	PR2	PR2	PR2	PR2	PR2	PR2	PR1
0.3	PR2	PR2	PR2	PR2	PR2	PR2	PR2	PR2	PR1	PR2	PR2	PR2	PR2	PR2	PR2	PR1
0.4	ML	ML	COR	COR	PR2	PR2	PR2	PR2	PR1	PR2	PR2	PR2	PR2	PR2	PR2	PR1
0.5	ML	COR	COR	COR	COR	PR2	PR2	PR2	PR1	PR1	PR2	PR2	PR2	PR2	PR1	PR1
0.6	COR	COR	COR	COR	COR	PR2	PR2	PR2	PR1	PR1	PR2	PR2	PR2	PR2	PR1	PR1
0.7	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	PR1
0.8	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.9	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.99	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR

When n =15 from Table 2, the best estimator is COR when $|\rho| \geq 0.5$ at all the levels of multicollinearity . Also, when $-0.1 \leq \rho \leq 0.4$ and $\lambda \geq 0.3$, the estimators based on using PC2, and very sparsely PC1, are best. At other instances, the best estimator is generally ML and sparsely COR; even though the two of them compete favorably together.

Table 2: The Best Estimator for Prediction at different level of Multicollinearity and Autocorrelation when n=15.

ρ	λ															
	-0.49	**-0.4**	**-0.3**	**-0.2**	**-0.1**	**0**	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**	**0.99**
-0.99	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.9	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.8	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.7	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.-6	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.5	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.4	COR	COR	COR	COR	COR	ML	ML	ML	COR	COR	COR	COR	COR	COR	COR	COR
-0.3	COR	COR	COR	ML	ML	ML	ML	ML	ML	ML	ML	ML	ML	COR	COR	COR
-0.2	COR	COR	ML	ML	ML	ML	ML	ML	ML	ML	ML	ML	ML	ML	ML	PR1
-0.1	COR	COR	ML	ML	ML	ML	ML	ML	PR2	PR2	PR2	PR2	PR2	PR2	PR2	PR1
0	COR	ML	ML	ML	ML	ML	ML	PR2	PR2	PR2	PR2	PR2	PR2	PR2	PR2	PR2
0.1	COR	ML	ML	ML	ML	ML	ML	PR2	PR2	PR2	PR2	PR2	PR2	PR2	PR2	PR2
0.2	COR	COR	ML	ML	ML	ML	ML	PR2	PR2	PR2	PR2	PR2	PR2	PR2	PR2	PR2
0.3	COR	COR	ML	ML	ML	ML	ML	ML	PR2	PR2	PR2	PR2	PR2	PR2	PR2	PR2
0.4	COR	COR	COR	COR	ML	ML	ML	ML	PR2	PR2	PR2	PR2	PR2	PR2	PR2	PR2
0.5	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.6	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.7	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.8	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.9	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.99	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR

From Table 3 when n =20, the best estimator is generally COR except when $-0.1 \le \rho \le 0.3$ and $\lambda \ge 0.2$. At these instances, the best estimator is generally estimators based on using PC2, and very sparsely PC1; even though the COR estimator occasionally perform equivalently.

Table 3: The Best Estimator for Prediction at different level of Multicollinearity and Autocorrelation when n=20.

ρ	λ															
	-0.49	-0.4	-0.3	-0.2	-0.1	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.99
-0.99	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.9	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.8	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.7	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.-6	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.5	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.4	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.3	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.2	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.1	COR	COR	COR	COR	COR	COR	COR	COR	PR2	PR2	PR2	PR2	PR2	PR2	PR2	PR2
0	COR	COR	COR	COR	COR	COR	COR	PR2	PR2	PR2	PR2	PR2	PR2	PR2	PR2	PR2
0.1	COR	COR	COR	COR	COR	COR	COR	PR2	PR2	PR2	PR2	PR2	PR2	PR2	PR2	PR2
0.2	COR	COR	COR	COR	COR	COR	COR	PR2	PR2	PR2	PR2	PR2	PR2	PR2	PR2	PR2
0.3	COR	COR	COR	COR	COR	COR	COR	COR	PR2	PR2	PR2	PR2	COR	COR	COR	PR1
0.4	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.5	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.6	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.7	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.8	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.9	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.99	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR

When n =30 from Table 4, the best estimator is COR when $|\rho| \ge 0.5$ at all the levels of multicollinearity. Also, when $-0.1 \le \rho \le 0.2$ and $\lambda \ge 0.1$, the estimators based on using PC2, and very sparsely PC1, are best. At other instances, the best estimator is generally ML and sparsely COR; even though the two of them compete favorably together.

Table 4: The Best Estimator for Prediction at different level of Multicollinearity and Autocorrelation when n=30.

ρ	λ															
	-0.49	-0.4	-0.3	-0.2	-0.1	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.99
-0.99	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.9	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.8	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.7	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.-6	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.5	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.4	COR	COR	COR	COR	COR	COR	ML	ML	ML	ML	ML	ML	ML	ML	COR	COR
-0.3	COR	COR	ML	ML	ML	ML	ML	ML	ML	ML	ML	ML	ML	ML	ML	ML
-0.2	COR	ML	ML	ML	ML	ML	ML	ML	ML	ML	ML	ML	ML	ML	ML	ML
-0.1	COR	ML	ML	ML	ML	ML	ML	PR2	PR2	PR2	PR2	PR2	PR2	PR2	PR2	PR2
0	ML	ML	ML	ML	ML	ML	PR2	PR2	PR2	PR2	PR2	PR2	PR2	PR2	PR2	PR2
0.1	ML	ML	ML	ML	ML	ML	PR2	PR2	PR2	PR2	PR2	PR2	PR2	PR2	PR2	PR2
0.2	COR	ML	ML	ML	ML	ML	ML	PR2	PR2	PR2	PR2	PR2	PR2	PR2	PR2	PR1
0.3	COR	ML	ML	ML	ML	ML	ML	ML	ML	ML	ML	ML	ML	ML	ML	ML
0.4	COR	COR	ML	ML	ML	ML	ML	ML	ML	ML	ML	ML	ML	ML	ML	COR
0.5	COR	COR	COR	COR	ML	ML	ML	ML	ML	ML	ML	COR	COR	COR	COR	COR
0.6	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.7	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.8	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.9	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.99	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR

From Table 5 when n =50, the best estimator is generally COR except when $|\lambda| \le 0.3$ and $\rho \to 1$. At these instances when $|\lambda| \le 0.1$, the best estimator is the estimator based on using PC2 and at other instances the ML estimator is best.

Table 5: The Best Estimator for Prediction at different level of Multicollinearity and Autocorrelation when n=50.

ρ	λ															
	-0.49	-0.4	-0.3	-0.2	-0.1	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.99
-0.99	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.9	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.8	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.7	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.-6	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.5	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.4	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.3	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	ML
-0.2	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	ML
-0.1	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	PR1
0	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	PR1
0.1	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	PR1
0.2	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	ML
0.3	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	ML
0.4	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.5	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.6	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.7	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.8	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.9	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.99	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR

Lastly from Table 6 when n =100, the best estimator is generally COR except in some instances when $|\rho| \leq 0.4$. and $\lambda \geq 0.5$. At these instances, the PC2 estimator is either best or competes with ML which is best when $0 \leq \rho \leq 0.1$ and $\lambda \geq 0.7$ At other instances, the ML estimator even though the COR estimator competes with it very well.

Table 6: The Best Estimator for Prediction at different level of Multicollinearity and Autocorrelation when n=100.

ρ	λ															
	-0.49	-0.4	-0.3	-0.2	-0.1	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.99
-0.99	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.9	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.8	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.7	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.-6	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.5	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.4	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	ML	ML
-0.3	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	ML	ML	ML	ML
-0.2	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	ML	ML	ML	ML	ML	ML
-0.1	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	ML	ML	ML	ML	ML	ML
0	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	ML	ML	PR2	PR2	PR2	PR2
0.1	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	ML	ML	ML	ML	PR2	PR2
0.2	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	ML	ML	ML	ML	ML	ML
0.3	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	ML	ML	ML	ML
0.4	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	ML	ML	ML
0.5	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.6	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.7	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.8	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.9	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.99	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR

4. Conclusion

The performances of estimators in and the choice of best estimator for prediction in regression analysis at different levels of autocorrelation, correlated uniform regresors and sample sizes have been critically examined through a Monte Carlo study in this paper. The performances of COR and ML estimators at each level of multicollinearity over the levels of autocorrelation are convex – like while that of the OLS and PC estimators are concave. Results further show that except when the sample size is small (n=10), the performances of the COR and ML estimators are generally best and asymptotically the same. However at low level of autocorrelation, the PC (PC2) estimator either performs better than or competes with the best estimator when $\lambda \leq -0.49$ and $\lambda \geq 0.1$. When the sample size is small (n =10), the COR estimator is best except when the autocorrelation level is low. At these instances, the PC2 estimator is either best or competes with the best estimator. Moreover, at low level of autocorrelation in all the sample sizes, the OLS estimator competes with the best estimator in all the levels of multicollinearity.

References

[1] Aitken, A. C. (1935): On least square and linear combinations of observations. Proceedings of Royal Statistical Society, Edinburgh, 55: 42–48.

[2] Adebayo, D. A. (2011): Estimators of Linear Regression Model with Autocorrelated Error term and Correlated Fixed Uniform Regressors". Unpublished M.Sc. Thesis. University of Ibadan, Nigeria.

[3] Ayinde, K. (2007): Equations to generate normal variates with desired intercorrelation matrix. International Journal of Statistics System. Vol. 2(2): 99–111.

[4] Ayinde,K .(2007): A Comparative Study of the Performances of the OLS and Some GLS Estimators When Stochastic Regressors Are both Collinear and Correlated with Error Terms. Journal of Mathematics and Statistics. 3(4): 196–200.

[5] Ayinde, K. (2008): Performances of Some Estimators of Linear Model when Stochastic Regressors are correlated with Autocorrelated Error Terms. European Journal of Scientific Research 20(3): 558-571.

[6] Ayinde, K. and Adegboye, O. S. (2010): Equations for generating normally distributed random variables with specified intercorrelation. Journal of Mathematical Sciences. 21(2):183–203.

[7] Ayinde,K. and Ipinyomi, R. A. (2007): A comparative study of the OLS and some GLS estimators when normally distributed regressors are stochastic. Trend in Applied Sciences Research, 2(4):354-359.

[8] Ayinde, K. and Iyaniwura, J.O. (2008): A Comparative study of the Performances of Some Estimators of Linear Model with fixed and stochastic regressors. Global Journal of Pure and Applied Sciences. 14(3): 363–369.

[9] Ayinde, K. and Oyejola, B. A. (2007): A comparative study of performances of OLS and some GLS estimators when stochastic regressors are correlated with error terms. Research Journal of Applied Sciences, 2: 215–220.

[10] Beach, C. M. and Mackinnon, J.S. (1978): A Maximum Likelihood Procedure regression with autocorrelated errors. Econometrica, 46: 51–57.

[11] Bock, M. E., Yancey, T. A. and Judge, G. G. (1973): Statistical consequences of preliminary test estimation in regression. Journal of the American Statistical Association, 60, 234–246.

[12] Chartterjee, S.,Hadi, A.S. and Price, B. (2000): Regression by Example. 3rd Edition. A Wiley- Interscience Publication, John Wiley and Sons.

[13] Chipman, J. S. (1979): Efficiency of Least Squares Estimation of Linear Trend when residuals are autocorrelated. Econometrica 47:115-127

[14] Cocharane, D. and Orcutt, G.H. (1949): Application of Least Square to relationship containing autocorrelated error terms. Journal of American Statistical Association, 44:32–61.

[15] Durbin, J. (1960): Estimation of Parameters in Time series Regression Models. Journal of Royal Statistical Society B, 22:139 -153.

[16] Fomby et al. (1984): Formy, T. B., Hill, R. C. and Johnson, S. R. (1984): Advance Econometric Methods. Springer-Verlag, New York, Berlin, Heidelberg, London, Paris, Tokyo.

[17] Helland I. S. (1988): On the structure of partial least squares regression. Communications in Statistics, Simulations and Computations, 17, 581 – 607.

[18] Helland I. S. (1990): Partial least squares regression and statistical methods. Scandinavian Journal of Statistics, 17, 97–114.

[19] Hoerl, A. E. (1962): Application of ridge analysis to regression problems. Chemical Engineering Progress, 58, 54–59.

[20] Hoerl, A. E. and Kennard, R. W. (1970): Ridge regression biased estimation for non-orthogonal problems. Technometrics, 8, 27–51.

[21] Hildreth, C. and Lu, J.Y. (1960): Demand relationships with autocorrelated disturbances. Michigan State University. Agricultural Experiment Statistical Bulletin, 276 East Lansing, Michigan.

[22] Iyaniwura, J.O. and Nwabueze, J. C. (2004): Estimators of Linear model with Autocorrelated error terms and trended independent variable. Journal of Nigeria Statistical Association, 17: 20 – 28.

[23] Iyaniwura, J.O. and Olaomi, J.O. (2006):Efficiency of the GLS estimators in linear regression with autocorrelated error terms which are correlated with regressors. Science Focus, 11:129–133.

[24] Johnston, J. (1984): Econometric Methods. 3rd Edition, New York, Mc,Graw Hill.

[25] Kramer, W. (1980): Finite sample efficiency of OLS in linear regression model with autocorrelated errors. Journal of American Statistical Association, 81:150–154.

[26] Kleiber, C. (2001): Finite sample efficiency of OLS in linear regression model with long memory disturbances.Economic Letters, 72: 131–136.

[27] Maddala, G. S. (2002): Introduction to Econometrics. 3rd Edition, John Willey and Sons Limited, England.
[28] Marquardt, D. W. (1970): Generalized inverse, Ridge Regression, Biased Linear Estimation and Non – linear Estimation. Technometrics 12, 591–612.
[29] Massy, W. F. (1965): Principal Component Regression in exploratory statistical research. Journal of the American Statistical Association, 60, 234–246.
[30] Naes, T. and Marten, H. (1988): Principal Component Regression in NIR analysis: View points,Background Details Selection of components. Journal of Chemometrics, 2, 155–167.
[31] Neter , J.and Wasserman, W. (1974): Applied Linear Model. Richard D. Irwin, Inc.
[32] Nwabueze, J.C. (2005a): Performances of Estimators of linear model with auto-correlated error terms when independent variable is normal. Journal of Nigerian Association of Mathematical Physics, 9:379–384.
[33] Nwabueze, J.C. (2005b): Performances of Estimators of linear model with auto-correlated error terms with exponential independent variable. Journal of Nigerian Association of Mathematical Physics, 9:385 – 388.
[34] Nwabueze, J.C. (2005C): Performances of Estimators of linear model with auto-correlated error terms when the independent variable is autoregressive. Global Journal of Pure and Applied Sciences, 11: 131-135.
[35] Paris, S. J. and Winstein, C. B. (1954): Trend estimators and serial correlation. Unpublished Cowles Commision, Discussion Paper, Chicago.
[36] Phatak, A. and Jony, S. D. (1997): The geometry of partial least squares. Journal of Chemometrics,11, 311–338.
[37] Rao, P.and Griliches, Z.(1969): Small sample properties of several two-stage regression methods in the context of autocorrelation error. Journal of American Statistical Association, 64:251–272.
[38] Schumann, E. (2009): Generating Correlated Uniform Variates. http://comisef.wikidot.com/tutorial:correlateduniformvariates
[39] Theil, H. (1971): Principle of Econometrics. New York, John Willey and Sons.
[40] Thornton, D. L. (1982): The appropriate autocorrelation transformation when autocorrelation process has a finite past. Federal Reserve Bank St. Louis, 82–102.