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Frequentist and Bayesian Estimation of Parameters of Linear Regression Model with Correlated Explanatory Variables

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Abstract

This paper addressed the popular issue of collinearity among explanatory variables in the context of a multiple linear regression analysis, and the parameter estimations of both the classical and the Bayesian methods. Five sample sizes: 10, 25, 50, 100 and 500 each replicated 10,000 times were simulated using Monte Carlo method. Four levelss of correlation $\rho = 0.0, 0.1, 0.5, and\ 0.9$ ***representing no correlation, weak correlation, moderate correlation and strong correlation were considered. The estimation techniques considered were; Ordinary Least Squares (OLS), Feasible Generalized Least Squares (FGLS) and Bayesian Methods. The performances of the estimators were evaluated using Absolute Bias (ABIAS) and Mean Square Error (MSE) of the estimates. In all cases considered, the Bayesian estimators had the best performance. It was consistently most efficient than the other estimators, namely OLS and FGLS.***

Keywords: Multicollinearity, Bayesian Estimation, Level of correlation, Feasible Generalized Least Squares, Mean Square Error

1.0 Introduction

Regression analysis is a central tool in applied statistics that aims to answer the general question of how two or more explanatory variables influence the outcome of the response variable. However, this influence can be seen to depend greatly on the degree of correlation existing among the explanatory variables. Such relationship among the explanatory variables is referred to as multicollinearity. Multicollinearity is one of several problems confronting researchers using regression analysis. To most of these researchers especially economists, the single equation least-squares regression model is a very popular and useful model which is tried and true. Its properties and limitations have been extensively studied and documented and are, for most part, well-known. Discussion of problems that arise as particular assumptions are violated are been extensively discussed in literature [1].

The history of multicollinearity dates back to 1934 when the multicollinearity concept was formulated to refer to the condition when the variables handled are under influence of two or more relationships. The term multicollinearity is used to denote the presence of linear relationships (or near linear relationships) among explanatory variables which results in a breakdown of the least squares procedures. If the explanatory variables are perfectly linearly correlated, the correlation coefficient for these variables is equal to unity [2]. When this happens, it is therefore normally impossible to interpret estimates of individual coefficients when the explanatory variables are mainly highly inter-correlated, no matter what the goal of multiple regression analysis is. In addition, review of the literature indicates that the multicollinearity problem has been handled in a variety of different ways [3]. Unfortunately, this problem arises often in practice, since many economic variables such as income, wealth, etc. are likely to be interrelated. Time series data are also likely to exhibit multicollinearity. Many economic series tend to move in the same direction (e.g., production, income and employment). When two or more independent variables are correlated, the statistical estimation techniques are incapable of sorting out the independent effects of each on the dependent variable.

While regression coefficients estimated using correlated independent variables are unbiased, they tend to have larger standard errors than they would have in the absence of multicollinearity.

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This in turn means that the t ratios will be smaller. Thus it is likely that the regression coefficients will not be significant than in the case where no multicollinearity plagues the data. In essence, there is less precision associated with the estimated coefficients [4].
Multicollinearity has been indicated to weaken accurate inference through its effect on the standard errors of the individual parameter estimates, increase variance of estimators, yield high coefficient of determination (R^2), leads to wider confidence intervals as well as lower test statistics values (in absolute value) in significant tests. Similarly, this results in parameter estimates with incorrect signs and unlikely magnitudes making it more difficult to specify the correct model [5-7].
Several estimators have been used in estimating the parameters of a linear model in the presence of correlated explanatory variables with the OLS being the oldest and the most popular. The FGLS which is an OLS estimator of a transformed isomorphic model that works on the shortcomings of OLS has also been employed by some authors. Bayesian on the other hand, offers ways to attain a reasonable estimate of the model parameters due to the possibility of including some sort of prior knowledge about these parameters [8]. However, the majority of Bayesian inference problems according to [9] can be seen as evaluation of the expectation of a function U(0) of interest under the parameter.
Multicollinearity is probably present in all regression analysis, since the independent variables are unlikely to be totally uncorrelated. Thus whether or not multicollinearity is a problem depends on the degree of collinearity. The difficulty is that there is no statistical test that can determine whether or not it really is a problem. One method to search for the problem is to look for "high" correlation coefficients between the variables included in a regression equation. Even then, however, this approach is not foolproof, since multicollinearity also exists if linear combinations of variables are used in a regression equation. There is no single preferable technique for overcoming muticollinearity, since the problem is due to the form of the data [4].
Collinearity among explanatory variables is one feature of the data that is directly related to the amount of information provided by the sample. When the sample is not informative enough to lead to significant conclusions, the only potential solution is to introduce more information. Although classical method rejects outright subjective information, it is not surprising that the search for operational solutions within its framework has failed to produce accepted techniques to combat multicollinearity. Hence, this study demonstrated that there is in fact an appropriate place for subjective information especially when the regressors are correlated. It seems more useful to speak in terms of the multicollinearity problem's severity rather than its existence. A case of perfect multicollinearity is rare, as is a zero correlation among explanatory variables (X's). Accordingly, multicollinearity will be defined here in terms of departures from independence, or from non-correlation, of the X's with one another. The major objective of this study is to compare the asymptotic behaviours of classical and Bayesian estimators at different levels of correlation among the explanatory variables.

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2.1 Material and Methods

In observational studies, the data generated by uncontrolled mechanisms may be subject to biases not present in controlled experiments. The most common problem is interrelationships among the independent variables that hinder precise identification of their separate effects. In such circumstances, regression parameters will tend to exhibit large sampling variances, perhaps leading to incorrect inferences regarding their significance, and there will be high correlations between parameters. Possible solutions to multicollinearity include the introduction of extra information, for example via prior restrictions on the parameters based on subject matter knowledge; the multivariate reduction of the set of covariates (e.g. by principal components analysis) to a smaller set of uncorrelated predictors; ridge regression [10], in which the parameters are a function of a shrinkage parameter k>0, with least squares estimate
Given the following model for multiple linear regression:

$$y_i = \beta_0 + \beta_1 x_{i1} + \sum_{j=2}^{k} \beta_j x_{ij} + \varepsilon_i \qquad \varepsilon_i \sim N(0, \sigma^2) \qquad \dots (2.1)$$

where $i = 1,2,\dots,n$

2.2 Effect of Multicollinearity on Sample Size

The effect of collinearity on parameter estimates is relative to the sample size. Intuitively, both collineaerity and sample size may be viewed as two very similar factors that determine the variability in the sample, which is the primary source of information offered by the data for statistical inference. Hence, the extent of either effect (high collinearity or small sample size) on individual parameter inference must be interpreted accordingly. From a purely classical objectivist perspective that obstinately refuses all prior information in statistical inference, this claim is undisputable. That is, one certainly cannot commit inferential exclusivity to a set of data, and upon receiving vague inference from the data, dismiss this vagueness on the grounds that the data is poorly conditioned. Hence, this study also investigates the effect of varying correlation among the regressors when sample sizes are 10, 25, 50, 100 and 500 respectively.

2.3 Detecting the Problem

A natural starting point to detecting multicollinearity problem is to look at the correlations between predictors. Perhaps there are only two predictors, this is sufficient enough to detect any problem with collinearity: if the correlation between the two predictors is zero then there is no problem. If the correlation is low then collinearity is probably just a minor nuisance - but will still reduce statistical power (meaning that there is less possibility to detect an effect and the effect will be measured less accurately). However with a larger correlation, there is more serious problem [11,12]. Furthermore with more than two predictors, the correlations between predictors can be misleading. Even if they are all very low (and unless they are exactly zero) they could conceal important multicollinearity problems. This will happen if the predictor's correlations do not overlap - and thus they have a cumulative effect. Working out the severity of the multicollinearity problems is not that easy and it is generally a good idea to use collinearity diagnostic. Fortunately there are a number of multicollinearity diagnotics that can help detect problems. Two of these diagnostic methods include tolerance and inflation factor (VIF).

2.4 Problems of the commonly used remedial Measures

In a situation where multicollinearity is detected, there are a number of ways in literature for dealing with it. However, the best remedy for multicollinearity is either to design a study to avoid it for instance, using an appropriate experimental design or increasing the sample size (which may not be feasible in some cases) to make your estimates sufficiently accurate. Increase in sample size decreases the effect of multicollinearity on the standard error (the larger the sample size, the smaller the standard error). If these methods are not feasible there are other options that may be helpful. One of which is to drop a predictor which may sometimes lead to a bigger problem such as misspecification error.

2.5 Model Estimation

The Classical Approach

The problem with the classical regression models in dealing with multicollinearity is that the standard errors associated with parameters estimates only reflect error due to sampling and there is no way to incorporate uncertainty that is associated with the model specification. Various classical estimation techniques have been used in literature on linear regression models. Of all these estimation techniques, only the methods used here shall be discussed, the Ordinary Least Squares (OLS) and the Feasible General Least Squares (FGLS).

A. Ordinary Least Squares (OLS)

Among all the various econometric methods that can be used to derive parameter estimates of econometric relationships, Ordinary Least Squares (OLS) stands on top priority list. It seeks the minimization of the sum of squares deviation of the actual observations on a variable from the values that would be obtained based on the regression equation. The ordinary least squares equation is given as:

$$y = X\beta + e \qquad e \sim N(0, \sigma^2) \qquad \ldots (2.2)$$

The OLS is known to be biased and inconsistent when endogenous variables appear as regressors in an equation and may be inefficient. Thus the OLS estimator for estimating β is given as:

$$\hat{\beta}_{ols} = (x^1x)^{-1}x^1y \qquad \ldots (2.3)$$

By decomposing β_{ols} estimator

$$\hat{\beta}_{ols} = (x^1x)^{-1}x^1y$$

The variance is given as

$$Var_{OLS}\left[\hat{\beta}|X\right] = \sigma^2(X'X)^{-1} \qquad \ldots (2.4)$$

B. Feasible Generalized Least Squares (FGLS)

The feasible generalised least squares is an ordinary least squares of the transformed variable that satisfies the standard least squares assumption. Feasible generalized Least Squares (FGLS) estimator is used when Σ is unknown. That is, the efficient estimation of β in feasible generalised least squares regression model does not require the knowledge of Ω - a positive definite symmetric matrix. In the estimation of FGLS, $\hat{\Omega}$ is used instead of Ω.

Thus the FGLS estimator for estimating β is given as:

$$\hat{\beta}_{fgls} = (x^l\Omega^{-1}x)^{-1}x^l\Omega^{-1}y \qquad \ldots (2.5)$$

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with

$$var_{fgls}[\widehat{\beta|x}] = \sigma^2(x'\Omega^{-1}x)^{-1} \qquad \ldots (2.6)$$

The Bayesian Approach

Several authors have worked on model estimation using Bayesian approach, [13-17]. Bayesian linear regression is an approach to linear regression in which the statistical analysis is undertaken within the context of Bayesian inference. It is commonly recommended as a means of dealing with multicollinearity. The thinking is similar to the recommendation for dealing with multicollinearity by increasing the sample size. If the analysis is based on more information then there should be no problem estimating the parameters more precisely [1,18-21]. In the Bayesian approach, we increase the information in the analysis by incorporating information about the prior beliefs about the parameter estimates as opposed to adding new data point [22-25]. When the regression model has errors that are normally distributed, and if a particular form of prior distribution is assumed, explicit results are available for the posterior probability distributions of the model's parameters. The Bayesian approach works on the principle of Bayes theorem that is:

$$p(\theta|y) = p(y|\theta)p(\theta)/p(y) \qquad \ldots (2.7)$$

However as $p(y)$ is independent of the data, this gives the posterior distribution as:

Posterior ∝ Likelihood x Prior

$$p(\theta|y) = p(y|\theta)p(\theta) \qquad \ldots (2.8)$$

By convention;

$p(\theta)$ - the prior distribution of θ (i.e. the distribution prior to observing the data y);

$p(y|\theta)$ – the likelihood function (i.e. the likelihood of observing the data given a particular parameter value θ)

$p(\theta|y)$ - the posterior probability distribution (i.e. the distribution of θ obtained after observing y and combining the information in the data with the information in the prior distribution);

$p(y)$ – the marginal density, is the sum(or integral) of $p(y|\theta)p(\theta)$ over all possible values of θ.

Prior probability density function $p(\theta)$

Non-informative prior is assumed for the parameters of the model. The idea behind the use of this prior (also known as flat, diffuse or locally-uniform prior) is to make inferences that are not greatly affected by central information or when external information is not available [26]. Two rules were suggested in [27] to serve as guide in choosing a prior distribution. The first one states that "if the parameter has any fixed value n, a finite range, or from -∞ to +∞, its prior probability should be taken as uniformly distributed". The second is that "if the parameter, by nature, can take any value from 0 to ∞, the prior probability of its logarithm should be taken as uniformly distributed.

The prior pdf is given as

$$p(\beta,\sigma^2) \propto \frac{1}{\sigma^2} \qquad \ldots (2.9)$$

$$p(\theta) = constant\ (prior\ pdf)$$

Likelihood Function $p(y|\theta)$

Based on the assumption from our model that $e \sim NIID(0,\Sigma)$, the likelihood function is given as;

$$p(y|\beta,\sigma^2) = \frac{1}{(2\pi)^{\frac{N}{2}}\sigma^N}\exp[-\frac{1}{2\sigma^2}\sum_{i=1}^{N}(y_i-\beta x_i)^2] \qquad \ldots (2.10)$$

Writing the likelihood in terms of precision gives

$$p(y|\beta,h) = \frac{1}{(2\pi)^{\frac{N}{2}}\sigma^N}\left\{h^{\frac{1}{2}}\exp[-\frac{h}{2}(\beta-\hat{\beta})^2\sum_{i=1}^{N}{x_i}^2]\right\}\left\{h^{\frac{v}{2}}\exp[-\frac{hv}{2s^{-2}}]\right\} \qquad \ldots (2.11)$$

where $h = \frac{1}{\sigma^2}$

Posterior PDF $p(\theta|y)$

Combining the prior pdf $p(\theta)$ with the likelihood function, gives a posterior distribution of the form;

$$p(\beta|y,h) = \frac{1}{(2\pi)^{\frac{N}{2}}\sigma^N}\left\{h^{\frac{1}{2}}\exp[-\frac{h}{2}(\beta-\hat{\beta})^2\sum_{i=1}^{N}{x_i}^2]\right\}\left\{h^{\frac{v}{2}}\exp[-\frac{hv}{2s^{-2}}]\right\}xp(\beta,\sigma^2) \propto \frac{1}{\sigma^2} \qquad \ldots (2.12)$$

The Bayesian approach in this research work is estimated using Monte Carlo simulation. Thus the posterior distribution of $\beta|\sigma^2$ is given as

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$$p\left(\beta|y,\frac{1}{\sigma^2}\right)\sim N(\beta_E, v_\beta\sigma^2) \qquad \ldots (2.13)$$

where θ represents the parameters i.e. β, σ^2 .

3.0 Methodology Review

Given that the number of observations is represented by n, the problem of estimating or predicting the value of a dependent variable y on the basis of a set of measurements taken on several independent variables $x_1, x_2, \ldots x_n$ is considered.

Generally, the model that relates observations and parameters may be written as;

$$(y|x,\beta,\sigma^2)\sim Normal(x\beta,\sigma_i{}^2) \qquad \ldots (3.1)$$

However, it is convenient to express the estimators by employing the following matrix notation:

$$y = x\beta + e \qquad e\sim N(0,\sigma^2) \qquad \ldots (3.2)$$

where;

$y = (y_1, y_2, \ldots y_n)^I$ is nx1 matrix of response variables

$e = (e_1, e_2, \ldots e_n)$ is nx1 matrix of residual variables

$x = (x_1, x_2, \ldots x_n)^I$ is the nxk design matrix of explanatory variables;

$\beta = kx1$ is a vector of regression coefficients.

$n =$ number of observations

Design of Monte Carlo Experiment

The experiment involved generating five explanatory variables from a uniform distribution for sample sizes 10, 25, 50, 100 and 500 out of which three (3) were made to correlate with specified correlation values of **0.0**, 0.1, 0.5 and 0.9 for no, weak, moderate and strong correlations respectively. The data was generated by arbitrarily fixing the following values for the true parameters.

$$\boldsymbol{\beta_1 = 0.1,\ \beta_2 = 0.99,\ \beta_3 = 0.13,\ \beta_4 = 1.00,\ \beta_5 = 0.11}$$

Likewise, the specific distributions for the predetermined variables were stated. Furthermore, the estimation of the already stated parameters of the model was carried out using both the classical and the Bayesian approaches.

4.0 Analysis and Interpretation of Results

The summary of results obtained for each sample size 10, 25, 50, 100 and 500 replicated 10,000 times under the degrees of correlation 0.0, 0.1, 0.5, 0.9 are presented in the subsequent tables.

The Table below shows the sum absolute bias and the MSE of the three estimators for each of the sample size replicated 10,000 times under the varying degrees of correlation.

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Table1: Absolute Biases across Correlation

RHO	0.0			0.1			0.5			0.9		
N	OLS	FGLS	BAYESIAN	OLS	FGLS	BAYESIAN	OLS	FGLS	BAYESIAN	OLS	FGLS	BAYESIAN
10	0.0132	0.0130	0.0130	0.0134	0.0133	0.0131	0.0148	0.0137	0.0134	0.0157	0.0145	0.0136
25	0.0129	0.0128	0.0126	0.0135	0.0132	0.0127	0.0149	0.0135	0.0130	0.0168	0.0147	0.0134
50	0.0126	0.0125	0.0125	0.0128	0.0127	0.0125	0.0136	0.0130	0.0128	0.0152	0.0138	0.0132
100	0.0114	0.0114	0.0114	0.0116	0.0115	0.0115	0.0125	0.0118	0.0116	0.0138	0.0136	0.0120
500	0.0100	0.0098	0.0098	0.0101	0.0099	0.0098	0.0112	0.0103	0.0100	0.0125	0.0110	0.0103

Table2: Absolute Biases across Sample Sizes

N	10			25			50			100			500		
RHO	OLS	FGLS	BAYESIAN	OLS	FGLS	BAYESIAN	OLS	FGLS	BAYESIAN	OLS	FGLS	BAYESIAN	OLS	FGLS	BAYESIAN
0.0	0.0132	0.0130	0.0130	0.0129	0.0128	0.0126	0.0126	0.0125	0.0125	0.0114	0.0114	0.0114	0.0100	0.0098	0.0098
0.1	0.0134	0.0133	0.0131	0.0135	0.0132	0.0127	0.0128	0.0127	0.0125	0.0116	0.0115	0.0115	0.0101	0.0099	0.0098
0.5	0.0148	0.0137	0.0134	0.0149	0.0135	0.0130	0.0136	0.0130	0.0128	0.0125	0.0118	0.0116	0.0112	0.0103	0.0100
0.9	0.0157	0.0145	0.0136	0.0168	0.0147	0.0134	0.0152	0.0138	0.0132	0.0138	0.0136	0.0120	0.0125	0.0110	0.0103

Table 3: MSE of the Estimators across Sample Sizes

N	10			25			50		
RHO	OLS	FGLS	BAYESIAN	OLS	FGLS	BAYESIAN	OLS	FGLS	BAYESIAN
0.0	0.000007156	0.000007032	0.000007028	0.000008382	0.000008280	0.000008280	0.000007116	0.000007070	0.000006966
0.1	0.000010656	0.000010374	0.000010178	0.000008462	0.000008180	0.000007662	0.000007188	0.000007094	0.000006842
0.5	0.000011884	0.000010542	0.000010468	0.000009602	0.000008038	0.000007508	0.000008700	0.000008020	0.000007820
0.9	0.000010418	0.000009006	0.000008048	0.000009972	0.000009374	0.000007926	0.000009912	0.000008208	0.000007508

N	100			500		
RHO	OLS	FGLS	BAYESIAN	OLS	FGLS	BAYESIAN
0.0	0.000006732	0.000006732	0.000006716	0.000004412	0.000004240	0.000004212
0.1	0.000005660	0.000005538	0.000005444	0.000005078	0.000004938	0.000004856
0.5	0.000006510	0.000005808	0.000005664	0.000005148	0.000004938	0.000004180
0.9	0.000007692	0.000007036	0.000005896	0.000006486	0.000005076	0.000004510

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Table 4: Performance of The OLS Estimator across Sample Sizes

	N = 10		N = 25		N = 50		N = 100		N = 500	
RHO	**ABIAS**	**MSE**	**ABIAS**	**MSE**	**ABIAS**	**MSE**	**ABIAS**	**MSE**	**ABIAS**	**MSE**
0.0	0.0132	0.000007156	0.0129	0.000008382	0.0126	0.000007116	0.0114	0.000006732	0.0100	0.000004412
0.1	0.0134	0.000010656	0.0135	0.000008462	0.0128	0.000007188	0.0116	0.000005660	0.0101	0.000005078
0.5	0.0148	0.000011884	0.0149	0.000009602	0.0136	0.000008700	0.0125	0.000006510	0.0112	0.000005148
0.9	0.0157	0.000010418	0.0168	0.000019972	0.0152	0.000009912	0.0138	0.000007692	0.0125	0.000006486

Table 5: Performance of the FGLS Estimator across Sample Sizes

	N = 10		N = 25		N = 50		N = 100		N = 500	
RHO	**ABIAS**	**MSE**	**ABIAS**	**MSE**	**ABIAS**	**MSE**	**ABIAS**	**MSE**	**ABIAS**	**MSE**
0.0	0.0130	0.000007032	0.0128	0.000008280	0.0125	0.000007070	0.0114	0.000006732	0.0098	0.000004240
0.1	0.0133	0.000010374	0.0132	0.000008180	0.0127	0.000007094	0.0115	0.000005538	0.0099	0.000004938
0.5	0.0137	0.000010542	0.0135	0.000008038	0.0130	0.000008020	0.0118	0.000005808	0.0103	0.000004938
0.9	0.0145	0.000009006	0.0147	0.000009374	0.0138	0.000008208	0.0136	0.000007036	0.0110	0.000005076

Table 6: Performance of the Bayesian Estimator across Sample Sizes

	N = 10		N = 25		N = 50		N = 100		N = 500	
RHO	**ABIAS**	**MSE**	**ABIAS**	**MSE**	**ABIAS**	**MSE**	**ABIAS**	**MSE**	**ABIAS**	**MSE**
0.0	0.0130	0.000007028	0.0126	0.000008280	0.0125	0.000006966	0.0114	0.000006716	0.0098	0.000004212
0.1	0.0131	0.000010178	0.0127	0.000007662	0.0125	0.000006842	0.0115	0.000005444	0.0098	0.000004856
0.5	0.0134	0.000010468	0.0130	0.000007508	0.0128	0.000007820	0.0116	0.000005664	0.0100	0.000004180
0.9	0.0136	0.000008048	0.0134	0.000007926	0.0132	0.000007508	0.0120	0.000005896	0.0103	0.000004510

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5.0 Discussion of Results

In Tables 1 and 2, the absolute bias (ABIAS) is used to judge the performances of the three techniques. The estimates of all the methods increase gradually as the level of correlation among the explanatory variables increases. However, the Bayesian method produced the smallest absolute biases for all the correlation levels considered. All the estimators showed an asymptotic behaviour, as their ABIAS estimates decrease as the sample size increases.
From Table 3, the MSE estimates of the methods increase consistently as the level of correlation among the variables increases. As the sample size increases, the MSE of all the methods also decrease except at N = 25. Again the Bayesian method outperformed the other estimators.
Tables 4, 5 and 6 give the summary of the performances of the estimators using both ABIAS and MSE across sample sizes.

6.0 Conclusion

The criteria used for comparison were majorly their absolute biases and mean square errors (MSE). From the analysis, the general performance of the Bayesian method was better compared to OLS and FGLS estimators. As the sample sizes increases, the bias reduced and the estimated parameters are approaching the specified/true parameters.
The performances of the OLS when there was no or a very weak correlation (i.e. $\rho = 0.0$ and 0.1) among the explanatory variables were not too far from each another with the exception of sample size 25 and mostly for the large samples. Consequently, OLS may best be used when there is no or very weak correlations for large samples however, defining how large is, is another bone of contention.
Furthermore, from the performances of FGLS, it can be best used when there is no correlation, weak correlation or moderate correlation among explanatory variables.
The performance of the Bayesian method is remarkable even for small samples cases n<50. To a very large extent, the Bayesian method is less sensitive to multicollinearity in estimating the parameters of linear regression models in the presence of correlated explanatory variables compared to the considered classical approaches. The Bayesian approach is asymptotically consistent in estimating the parameters of linear regression models in the presence of correlated explanatory variables. This is a strong point for the Bayesian method. Consequently, to determine the contribution of each explanatory variable for any sample size at any level of correlation, the Bayesian method should be preferred over the classical methods.
Thus, we would conclude on the basis of this analysis and statistical investigation that the Bayesian approach is better and should be preferred in estimating the parameters of linear model in the presence of correlated explanatory variables.

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