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# An Application of Bayesian Dynamic Linear Model to Okun's Law

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**Abstract**

Many authors have used dynamic time series regression models to analyse Okun's law. This type of models often require first differencing the dependent and independent variables, as well as investigating the maximum lag length required for the model to be efficient. In this paper, we propose a straight-forward time-varying parameter state space model for analyzing Okun's law. In particular, as a case study, we investigate the validity and stability of Okuns law using a Bayesian Dynamic Linear Model which implicitly describes the time-varying relationship between Gross Domestic Product (GDP) and unemployment rate of a major economy in Africa for three decades. The time-varying parameters of this model are estimated via a modified recursive forward filtering, backward sampling algorithm. We find that Okuns law exhibited structural instability in Nigeria in the period 1970-2011, with the sensitivity of unemployment rate to movements in output growth loosing stability over time, which may have been a contributor to her recent economic decline.

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# 1 Introduction

Over the years, Okuns law, the negative relationship between movements of the unemployment rate and real economic activity, has been widely accepted by many researchers as an empirical regularity in macroeconomics. However, it is important to examine whether the relationship between unemployment and economic growth is stable over time or not. A great deal of theoretical and empirical studies have been devoted to both unemployment and Gross Domestic Product (GDP), and how the two variables are related since the first empirical investigation of the relationship between output gap and employment gap by [1]. Okuns original work states that a one percent point reduction in unemployment rate would increase output by approximately 3 percent. The major implication of this is that, to avoid the wastage associated with unemployment or economic slack, the economy must continually expand [2]. Okun's law simply portrays negative relationship between unemployment and GDP. Nigeria, being the second largest economy in the continent of Africa and one of the top largest economies in the world recorded considerable decline in GDP in the last two quarters of 2016. In fact, the Nigerian economy can presently be said to be in a state of recession while the prevailing high level of unemployment remains quite worrisome and has been a source of deep concern to policy makers and government, in particular. Many studies have discovered that unemployment and GDP have been moving in the same direction in Nigeria, which appears to be at variance with Okuns law. This therefore provokes this investigation about the stability and reliability of Okuns law in Nigeria, while attempting to study the time-varying parameter structure. It is important for policymakers to understand the magnitude and direction of the changes in Okun's law and differences over time. The goal of this paper, therefore, is to contribute to the literature on Okuns law by assessing its stability in Nigeria over time using a Time-Varying Parameter (TVP) model. Hence, our work is a fresh attempt carried out to estimate Okuns coefficient for Nigerian economy and to test

the validity of Okuns Hypothesis with the aim of making recommendations to macroeconomic policy makers in Nigeria.

## 2 Brief Literature

The empirical study on the relationship between output gap and employment gap by Okun (1962) has been the bedrock and foundation of several other empirical studies in literature. This relationship is regarded as the rule of thumb among policymakers as it determines the direction and description of macroeconomic policy implications [3, 4, 5]. Determination of empirical implications of Okun's law was re- fuelled by simultaneous occurrence of low level of unemployment and inflation( a phenomenon which we term astagflation) in the 1990s mostly in advanced western societies [6]. Empirical efforts into this were re-invigorated after the slowing of both economic growth and unemployment in the United State's economy as reported by [7]. A lot of empirical studies have investigated the sign and the magnitude of Okuns coefficient adopting different specifications and for different countries. [8] investigated the stability of Okuns law for Canada and the United States using a time-varying approach. Time variation was modeled as driftless random walks and estimated using the median unbiased estimator approach developed by [9]. They found that Okuns law exhibits structural instability in both countries, with the sensitivity of the unemployment rate to movements in output growth increasing recently over time in both Canada and the United States. [10] examined the nature of relationship between output and unemployment in the Malaysian economy. The empirical results show that there was an inverse relationship between output and unemployment. Also, [11] investigated Okuns law for Spanish regions using data for the period 1980-2004. His finding established the existence of Okuns law for most of the regions and for the economy as a whole. However, the magnitude of Okuns coefficient differed for various regions due to regional productivity differentials. Irfan Lal et al. (2010) estimated the Okuns coefficient, and checked the validity of Okuns law in some Asian countries by using the time series of annual data during the period 1980-2006 and applying co-integration technique and error correction mechanism (ECM), they argued that Okuns law interpretation may not be applicable and also the principle of

Natural Inflation Rate of Unemployment (NAIRU) does not hold its validity in some Asian developing countries. Their study agreed with the study by [12] in Nigeria who also established positive relationship between output changes and unemployment rate. [7] examined the stability of the Okun relationship and found that there is an asymmetric behavior with the output effect being more severe upon unemployment when the economy is in recession. In the same spirit, [4] argued that it is essential to distinguish between short-term and long-term effects when examining the Okun relationship, whereas they find that the short-run effects are in general, weaker than the long-term ones. The paper of [13] addressed the estimation of the relationship between unemployment and output for the case of Romania. He used the Bayesian linear regression model to assess the sensitivity of results to three prior distributions and his results indicated an Okun coefficient around -0.20 in line with other researches undertaken for Romania. He also suggested that similar studies should be done for other emerging economies. [14] noticed a significant increase in unemployment after Great Depression, which is larger than the one Okuns coefficient postulates and they argued that this may be attributed to the increased rise in labor productivity. However, [15] could not substantiate this evidence as they failed to find evidence that the Great Recession cause substantial changes in Okuns coefficient.

Similarly, [16] estimated the Okuns coefficient, and checked the validity of Okuns law in Nigeria, using annual time series data during the period 1980-2008 by means of Engle-Granger co-integration test and fully modified Ordinary Least Squares (OLS). They found a positive coefficient in the regression, and therefore argue that Okuns law interpretation is not applicable to Nigeria. However, in a more recent study by [17], they investigated the relationship between unemployment and GDP growth in 9 Arab countries using Pooled Generalized Least Square (PGLS). They found that 1 per cent increase in economic Growth will decrease the unemployment rate by 0.16 per cent. [18] also tested the validity of Okuns law in Nigeria using Error Correction Model (ECM) and Johasen cointegration test. Empirical findings show that there is both short and the long run relationship between unemployment rate and output growth in Nigeria.

Recent investigations into the Okuns law have centered more on the the stability of Okuns law i.e whether the effect of output upon unemployment

is different depending on the phase of the business cycle of the economy. By implication, the relationships between output and unemployment exhibit instability over time and as a result, empirical investigations of Okun law should center on time variation dimension rather than estimating just a coefficient to overall time horizon. Some scholars are of the opinion that recession could result in serious deviation and instability in the observed Okun's relationship [5, 10, 17, 19, 20, 21], mohd2007, beaton2010, owyang2012,abdul2014.

From the foregoing, it is clear that empirical studies applying time-varying parameter estimation approach to quantitative verification of Okun law is quite extant in literature, and as a matter of fact, no study could be found in Nigeria on time variation of Okun law using a state space model.

This present study examines the impact of unemployment on the Nigerian Economic growth as well as investigates the sensitivity of output to change in unemployment rates in Nigeria over time using a Bayesian Dynamic Linear Model (BDLM), which adopts a time-variation approach.

# 3 Model Specification and Methodology

We specify a Bayesian dynamic linear regression model with time-varying parameters to assess the time varying relationship between real Gross Domestic Product (GDP) and unemployment for the Nigerian economy. Our model specification takes the following form:

$$y_t = \alpha_t + x_t'\theta_t + v_t \qquad v_t \sim N(0, V) \tag{1}$$

$$\alpha_{t+1} = \alpha_t + q_{t1} \tag{2}$$

$$\theta_{t+1} = \theta_t + q_{t2} \tag{3}$$

$$\theta_0 \sim N(m_0, C_0)$$

where

$$q_t = (q_{t1}, q_{t2})' \sim N(0, W_t)$$

Equation (1) is known as the observation equation while equations (2) and (3) are the evolution equations. $y_t$ is a scalar of response; $x_t$ and $z_t$ are $k \times 1$ and $p \times 1$ vectors of covariates respectively; $\beta_t$ is a $k \times 1$ vector of time varying-intercept coefficients; $\theta_t$ is a $p \times 1$ vector of time-varying slope parameters.

We assume that all $v_t$'s are independent from the $w_t$'s. Since each parameter at time $t$ only depends on results from time $t-1$, the time-varying slope parameters $\theta_t$ is formulated to constitute a first order random walk process in equation 3 to allow for both temporary and permanent shifts in the parameters. The drifting coefficient is meant to capture a possible non-linearity such as a sudden jump or structural break. The disturbance of observational variance denoted by $v_t$ is assumed to follow the normal distribution with time-invariant variance $V$.

In our model, the response $y_t$ is the annual GDP of Nigeria from 1970 to 2011. $x_t$ represents unemployment rates measured concurrently with annual GDP . $\beta_t$ represents dynamic intercept terms. $\theta_t$ are time- varying Okun's coefficients which describes the relationship between the regressor and the response at each time $t$.

## 3.1 Bayesian Estimation of the Model Parameters

Due to the Markovian structure of the parameters $\theta_t$, we estimate the model by the method of Kalman filter [**?**, **?**, **?**] by computing the predictive and filtering distributions of $\theta_t$ recursively starting from $\theta_0 \sim N(m_0, C_0)$. The Kalman filter calculates the mean and variance of the unobserved state $\theta_t$, given the observations. It is a recursive algorithm i.e the current best estimate is updated whenever a new observation is obtained.

Let $\theta_{t-1}|y_{1:t-1} \sim N(m_{t-1}, C_{t-1})$, where $y_{1:t-1}$ denotes all observations up to time $t-1$. The one-step-ahead predictive density $\theta_t|y_{1:t-1}$ is Gaussian with parameters:

$$E(\theta_t|y_{1:t-1}) = m_{t-1} \equiv A_t \tag{4}$$

$$Var(\theta_t|y_{1:t-1}) = C_{t-1} + W_t \equiv R_t \tag{5}$$

The one-step-ahead predictive density of $y_t|y_{1:t-1}$ is Gaussian with parameters:

$$f_t = E(y_t|y_{1:t-1}) = x_t A_t \tag{6}$$

$$Q_t = Var(y_t|y_{1:t-1}) = x_t R_t x_t^{'} + V \tag{7}$$

The filtering density of $\theta_t$ given $y_{1:t}$ is Gaussian with parameters:

$$m_t = E(\theta_t|y_{1:t}) = A_t + R_t x_t^{'} Q_t^{-1} e_t \tag{8}$$

$$C_t = Var(\theta_t|y_{1:t}) = R_t - R_t x_t^{'} Q_t^{-1} x_t R_t \tag{9}$$

$$\tag{10}$$

where $e_t = y_t - f_t$ is the forecast error.

For the Kalman filter to run, it is necessary to know $V$ and $W_t$. Estimation of $V$ is done using the Gibbs sampler described in section 3.2. We use discount factors to estimate $W_t$ as in [22, 25].

The idea behind discount factors is to represent $W_t$ as a proportion of filtering distribution variance $C_t$. If $C_t$ is large then there is high uncertainty in moving from $\theta_{t-1}$ to $\theta_t$. Since $W_t$ represents this uncertainty, it is natural to model it as proportional to $C_t$. Thus, we select a discounting parameter $\lambda$ and set

$$W_t = \frac{1-\lambda}{\lambda} C_{t-1} \tag{11}$$

This method has the advantage of giving a natural interpretation to $W_t$ while also allowing it to vary through time to model changes in volatility while considering various granularities of $\lambda$ which ranges from $0 < \lambda < 1$.

## 3.2 Recursive Forward Filtering Backward Sampling(RFFBS) Algorithm and Gibbs Sampler

In order to estimate $V$ we use Gibbs sampling. This requires us to draw samples from $V|\theta$ as well as from $\theta|V$. The latter draw is performed using the Forward Filtering Backwards Sampling (FFBS) algorithm of [**?**, **?**].

We begin by initializing $V^{(0)}$ and running the Kalman filter on the data using these initial values for V.

1. We denote $p(\theta_0, ..., \theta_T|D_T) = \prod_{t=0}^{T} p(\theta_t|\theta_{t+1}, ..., \theta_T, D_T)$

2. We then sample from $p(\theta_T|D_T)$ using the filtering density above.

3. By the Markov property,

$$p(\theta_t|\theta_{t+1}, ..., \theta_T, D_T) = p(\theta_t|\theta_{t+1}, D_T)$$

where

$$D_T = (y_1, y_2, y_3, ..., y_T)$$

4. We then proceed inductively until we have a complete sample from $p(\theta_0, ..., \theta_T|D_T)$. Since we proceed from t = T to t = 0, this is called backwards sampling.

To sample from $V|\theta$ we impose a gamma prior on $V^{-1}$ and derive the posterior hyperparameters. Let $V^{-1} \sim Gamma(a_0, b_0)$, then

$$V^{-1}|\theta \sim Gamma(a_0 + \frac{T}{2}, b_0 + \frac{1}{2}\sum_{t=1}^{T}(y_t - x_t\theta_t)^2)$$

The Gibbs sampler proceeds as follows.
First, initialize $V^{(0)} \sim Gamma(a_0, b_0)$. Then, for i = 1, . . . , M,

1. Sample $\theta^{(i)}$ using FFBS.

2. Sample $V^{-1(i)}|\theta^{(i)} \sim Gamma(a_0 + \frac{T}{2}, b_0 + \frac{1}{2}\sum_{t=1}^{T}(y_t - x_t\theta_t^{(i)})^2)$

This Gibbs sampler was also run for $W_t$ determined from a given granularity of $\lambda$ as mentioned above. We used M = 12,000 with a burn-in period of 2,000. Convergence was quite quick as a result of the recursive forward filtering, backward sampling algorithm embedded within the Gibbs sampler.

# 4 Empirical Analyses and Results

The methodology discussed in the previous section is used to estimate the validity and reliability of Okuns law for Nigeria in this section.

## 4.1 Data

The data set used in this study comprises the unemployment rate and (log) real Gross Domestic Product(GDP) of Nigeria obtained from the Central Bank of Nigeria Statistical Bulletin 2012 . The data spans from 1970-2011.

Based on the visual consideration of the economic indicators used, it appears that the Nigerian economy was stable during the years immediately after

independence and into the oil boom years. Nigerian GDP rose strongly from 2003 probably because of growth in non-oil sectors as a result of increased money stock due to decreased lending rate.To avoid spurious regression, we adjust GDP by taking logarithm of GDP. Taking logarithm enables us to uncover the real growth in the economic variable, if any. It also helps to stabilize the variance of random or seasonal fluctuations and/or highlight cyclical patterns in the data. Figures 1 and 2 are the plots of GDP and log of GDP respectively, while Figure 3 is the time plot of unemployent rate. Y-axis depicts the number of years in each plot. UER stands for Unemployment Rate in Nigeria whose time plot is shown in Figure 4.

## 4.2 Discussion of Results

As can been seen from Figure 5 (which shows the time-varying estimates of Okun's slope coefficients), movement behaviour of unemployment and GDP were inconsistent with Okuns law in the early 70s. A positive relationship could be observed between the two variables. This however can be explained in the light of the civil war which the country went through towards the end of 1960s and in the early 1970s. High unemployment over this period could be explained by diversion of productive activities to fighting war as well as unfavourable and unstable macroeconomic environment which made investment unprofitable and could therefore not generate employment for the populace. The positive movement of GDP with this level of unemployment could be justified on the basis of heavy investment in military spending and security needed to fight the war during this period. From the early 70s towards the end of the 1980s, behaviour of unemployment and GDP were consistent with Okuns law. This could be due to the regime shifts in the country with okuns coefficients ranging between 0.00 to -0.04.Figure 5 depicts the dynamic regression of log GDP on unemployment rate which shows the predictive behaviour od GDP by unemployment, *ceteris paribus*. It is evident from the graph that there is relative instability in predictive power of unemployment in explaining GDP. However, for some periods like early 1970s, between 1980s-late 1990s and early periods of 2000s, unemployment seems better in predicting GDP in Nigeria.

According to Figure 6, a positive relationship was observed towards the

end of 1970s and early 1980s. However, from early 1980s, Okuns coefficient was negative, approaching zero until the turn of the millennium. From year 2000 onward, Okuns coefficient has been moving hapahazardly towards zero, showing that GDP movement in Nigeria could not be associated with meaningful productive employment opportunities as unemployment remains largely high in the country, which may be the associated with the present economic recession being experienced in Nigeria. The trace plot from our model do not show any particular sign of non-convergence as revealed in Figure 7.

# 5 Conclusion and Policy Recommendation

This study examines the reliability and stability of Okuns law in Nigeria using a time varying approach. The overriding picture that emerges from our investigation is that Okuns law in Nigeria has been largely unreliable and unstable, most especially in terms of magnitude. Okuns coefficient has been fluctuating in Nigeria, ranging between 0.8 to  0.4 as against stable value of -2 implied by Okuns law. Meanwhile, in terms of direction, behaviour of unemployment and GDP conformed with the law around mid 1970s and from early 1980s Okuns law exhibits high level of instability. Therefore, the study recommends that policy formulation and recommendation in accordance with the traditional western economic laws may not be of much help to developing countries. In other words, policy measures to curb unemployment in Nigeria and other developing countries should not flow from the traditional western laws as some of these laws could not conform to economic realities in developing countries, given certain structural rigidities that are so pervasive and inherent in developing countries.

The flunctuating nature of Okun's law in Nigeria discovered in this study has significant implications for development programmes and policies introduced by the government of Nigeria which should aim at declining unemployment rates while increasing economic growth rate. It was clearly seen for most time during the period under consideration that while unemployment was increasing, the economy was equally growing. This could be as a result of over dependence on oil as a major source of revenue to the nation. Hence, this study recommends that the new regime and economic policy makers in Nige-

ria should embrace benign policies and activities aimed at promoting economic growth in the country which is also geared towards promoting employment for the people, while ensuring equitable distribution of wealth and resources in Nigeria.

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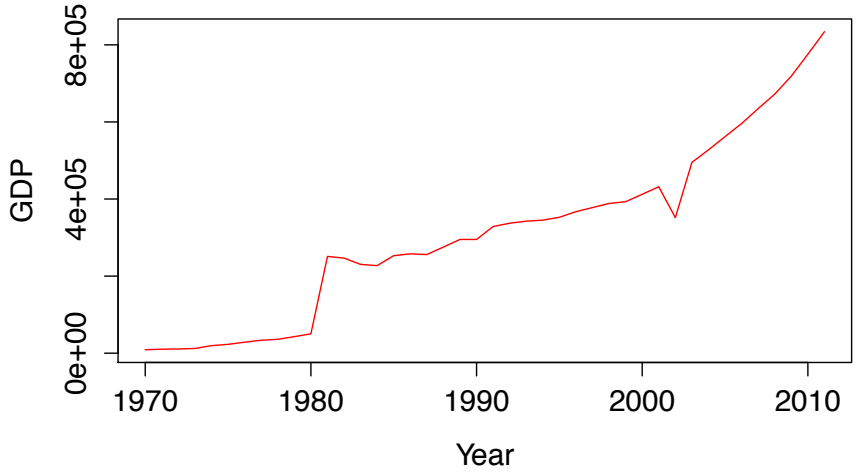


Figure 1: Nigerian GDP(1970-2011)

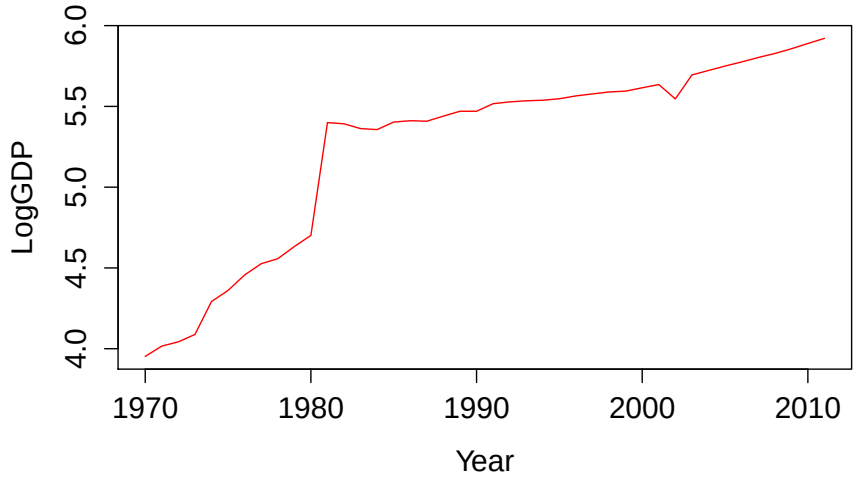


Figure 2: Log of GDP(1970-2011)

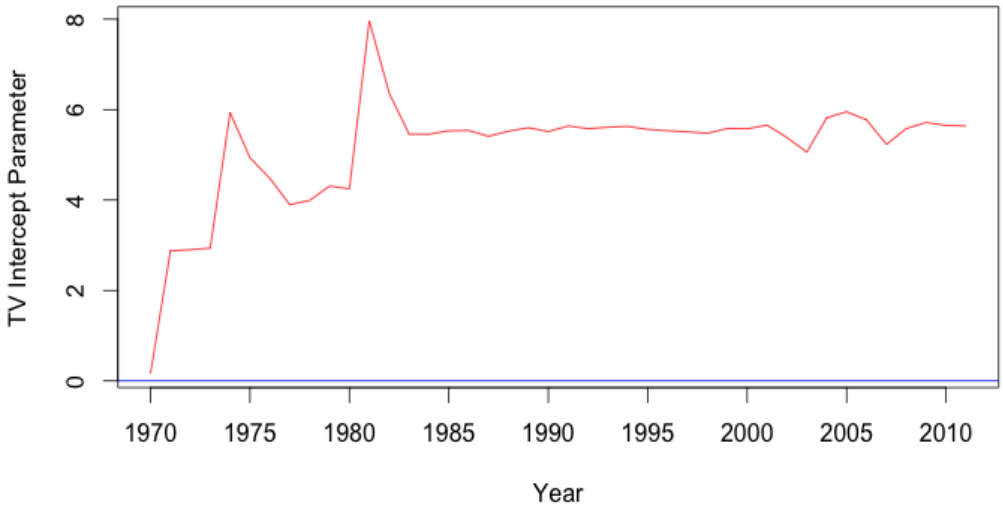


Figure 3: Time Varying Intercept Parameters(1970-2011)

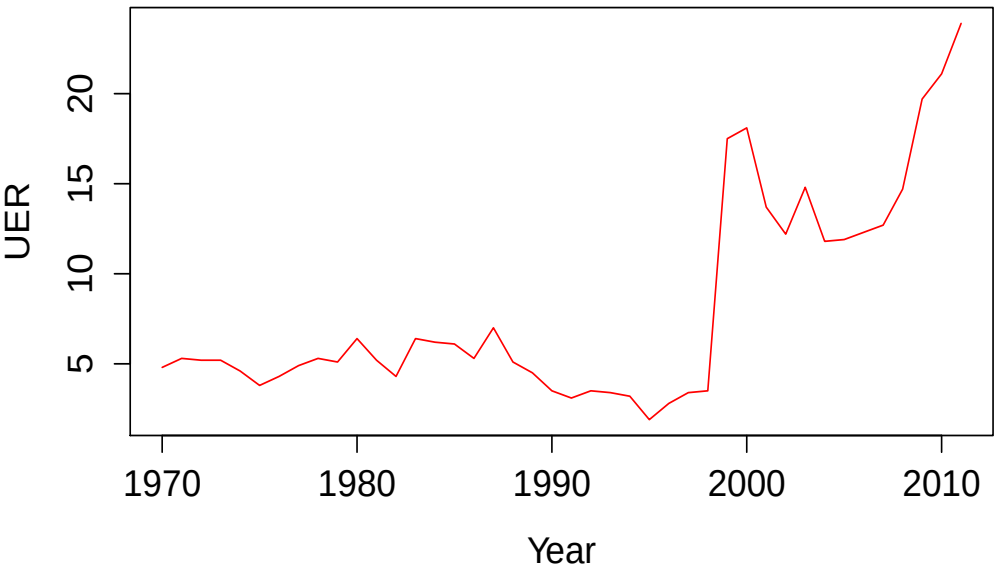


Figure 4: Unemployment Rate in Nigeria(1970-2011)

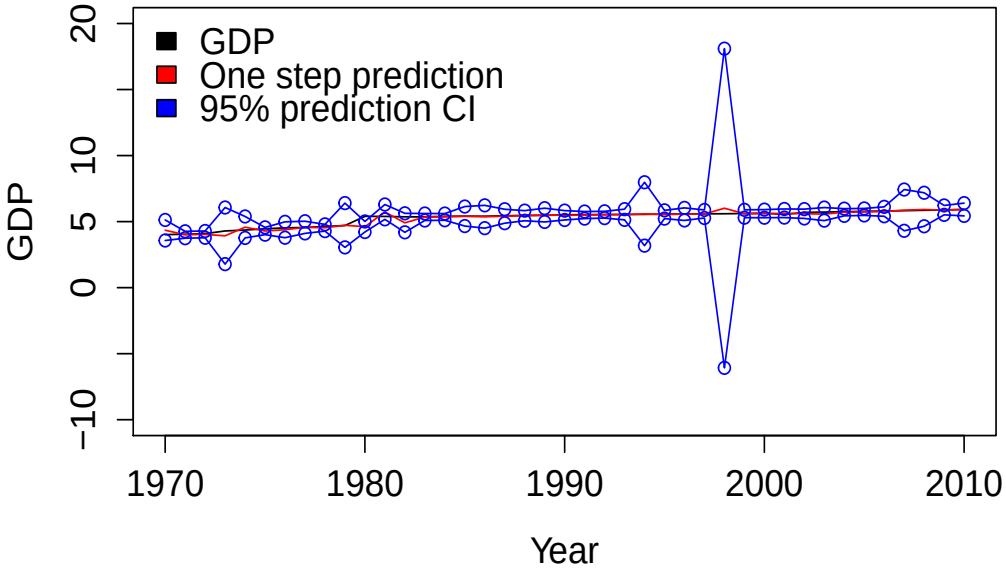


Figure 5: Dynamic Regression of Log GDP on Unemployment Rate

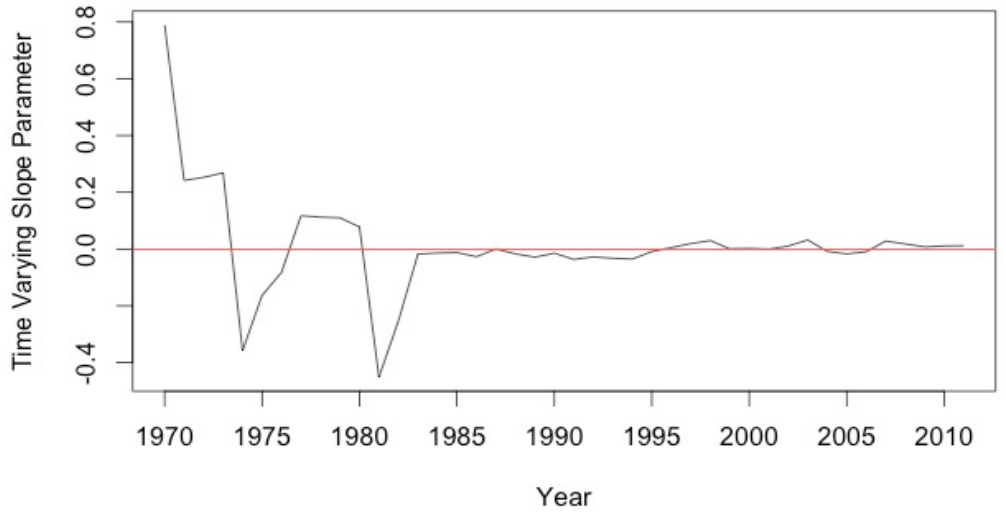


Figure 6: Time Varying Parameter Estimates

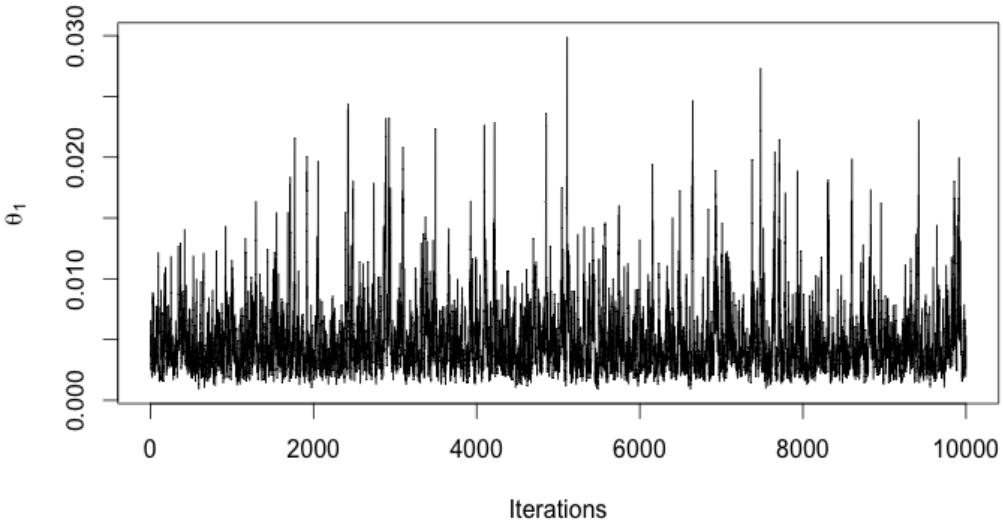


Figure 7: Trace Plot